Describing Chaotic Dynamics in Experimental Rayleigh-Bénard Convection Using Persistent Homology Theory

Jeff Tithof
Bala Suri
Sam Raben
Mike Schatz

Georgia Tech

Miro Kramar
Konstantin Mischaikow

RUTGERS

Mu Xu
Mark Paul

Virginia Tech

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Preview

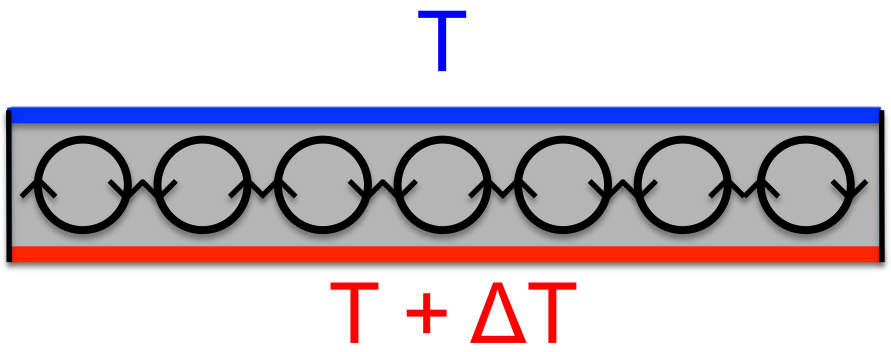
- Introduction / Previous Work
- Persistent Homology (with Examples)
- Experimental Setup and Data
- Results
- Future Work

Motivation

- Complex patterns in nature
- Challenge: extracting information, characterizing, and making predictions
- Apply new methods to analyze patterns in Rayleigh-Bénard Convection (RBC)
  - Plethora of complicated patterns
  - Physical Motivation
    - Earth's atmosphere, ocean, & mantle
    - Industrial applications
- Combined experimental and numerical study

Rayleigh-Bénard Convection

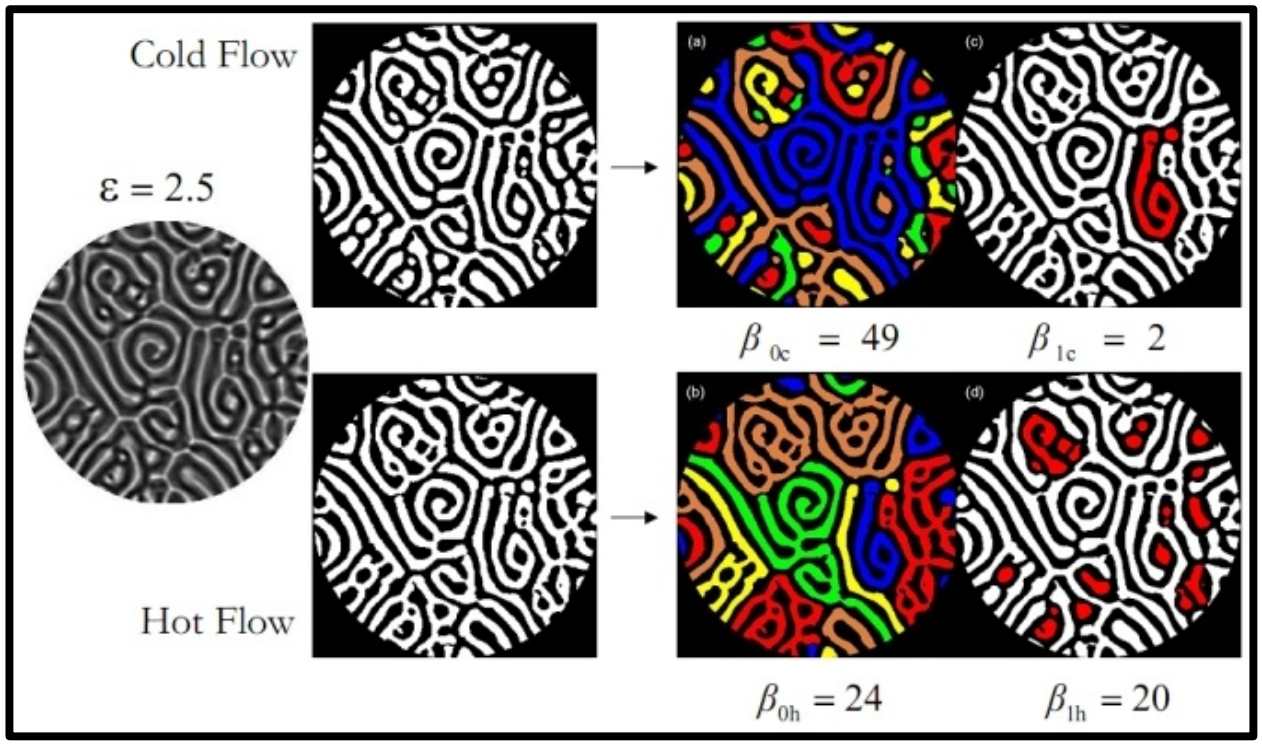
View From Side



View From Above

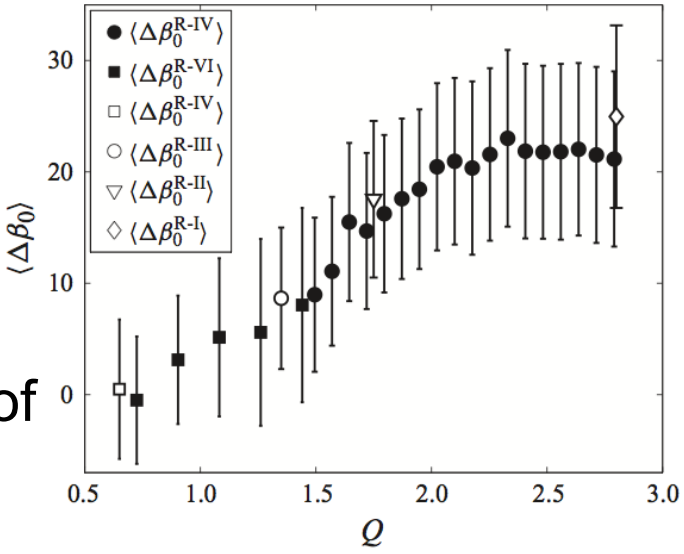
Computational Homology

- Measures global geometric properties
- Betti Numbers
  - β_0 - # distinct connected pieces
  - β_1 - # holes

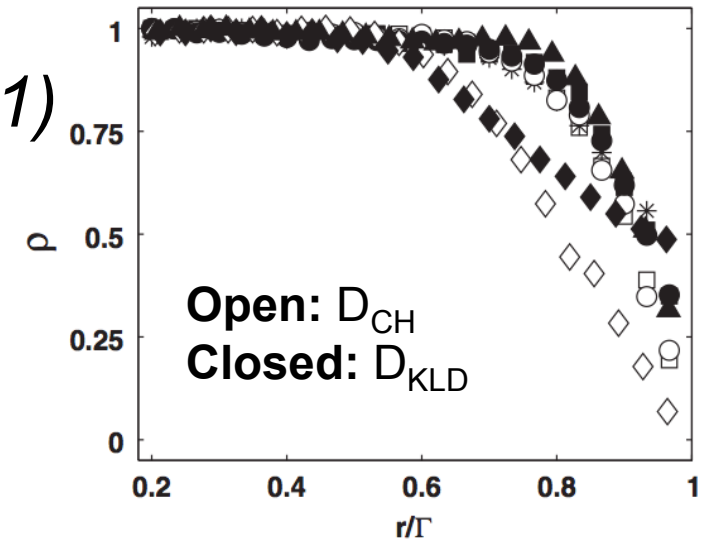


Previous Work

- *Phys. Fluids **19**, 117105 (2007) &*
  *J. Fluid Mech. **682**, 543-557 (2011)*
  - Betti numbers show breakdown of Oberbeck-Boussinesq Approx.
    - OB Approx – Ignores T-dependence of all fluid properties except density variation that causes buoyancy force

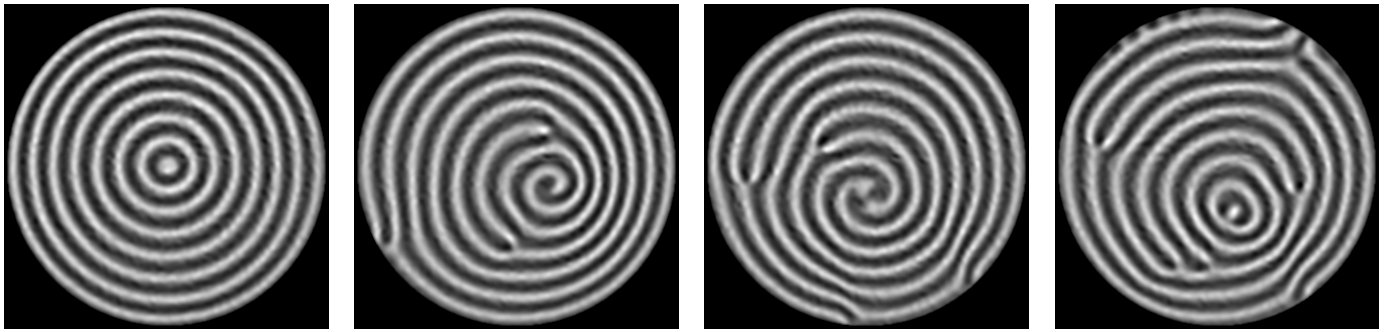


- *Phys. Rev. Lett. **107**, 034503 (2011)*
  - Dynamical DOF scale
  - Finite size effects
  - Comparison to KLD



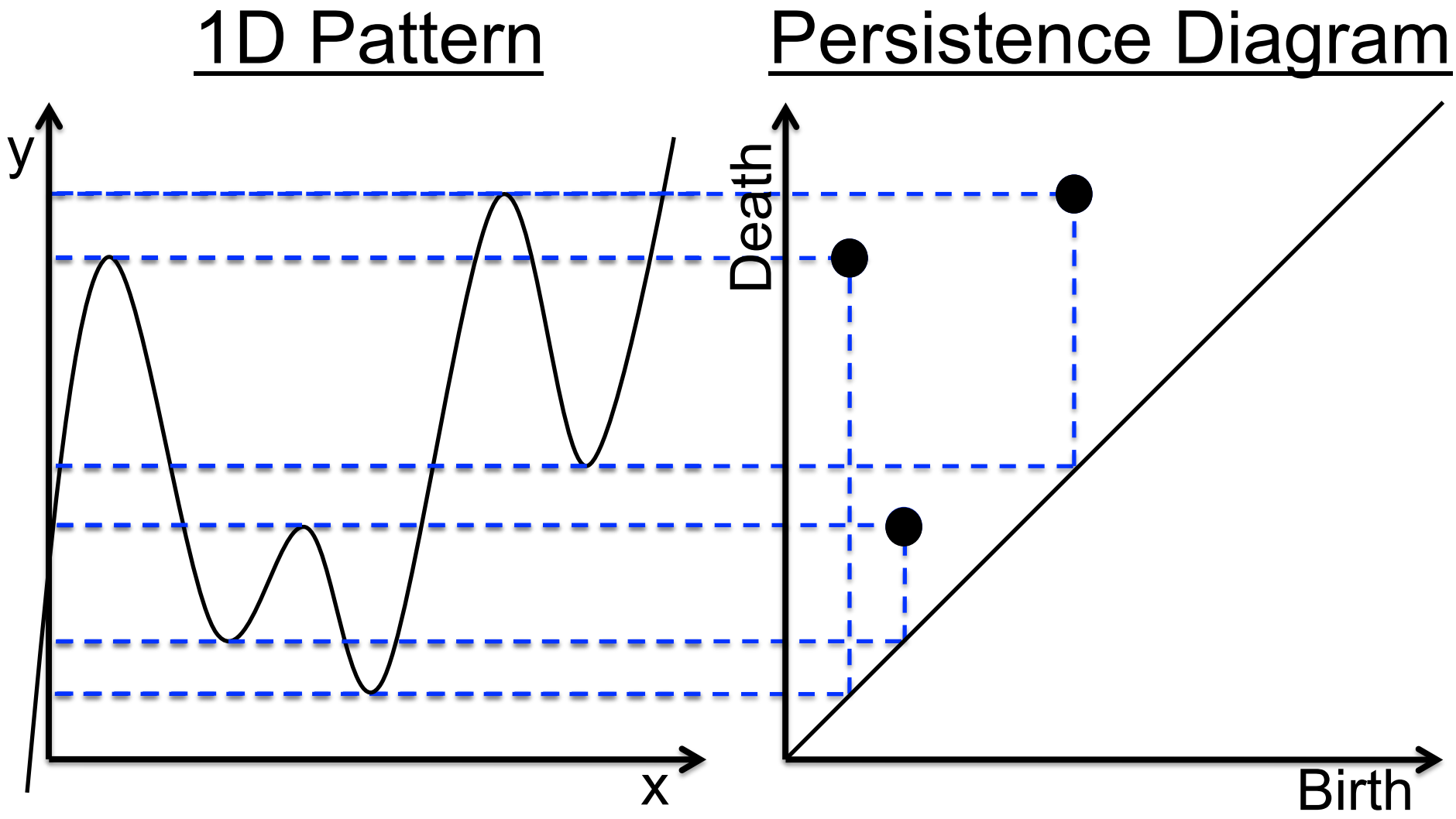
New Methods

- Persistent Homology
  - Study topological properties as we vary threshold
- Study time evolution in weakly chaotic regime



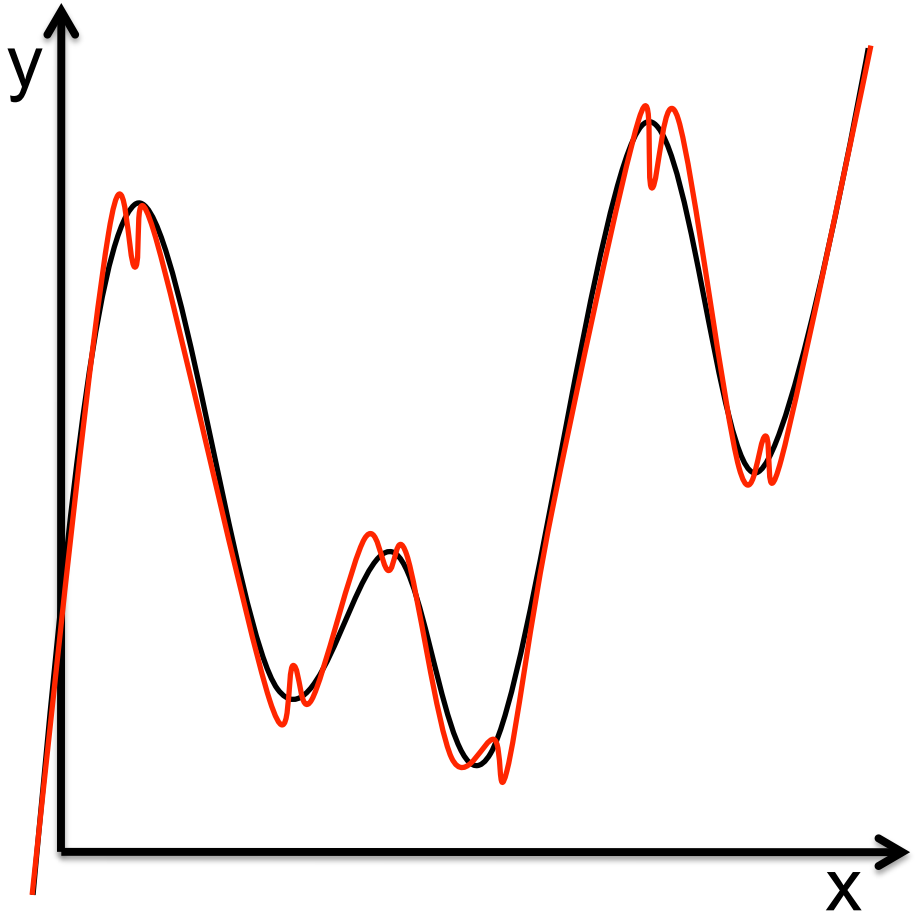
Royer et. al., PRE 70, 036313 (2004)

Persistent Homology

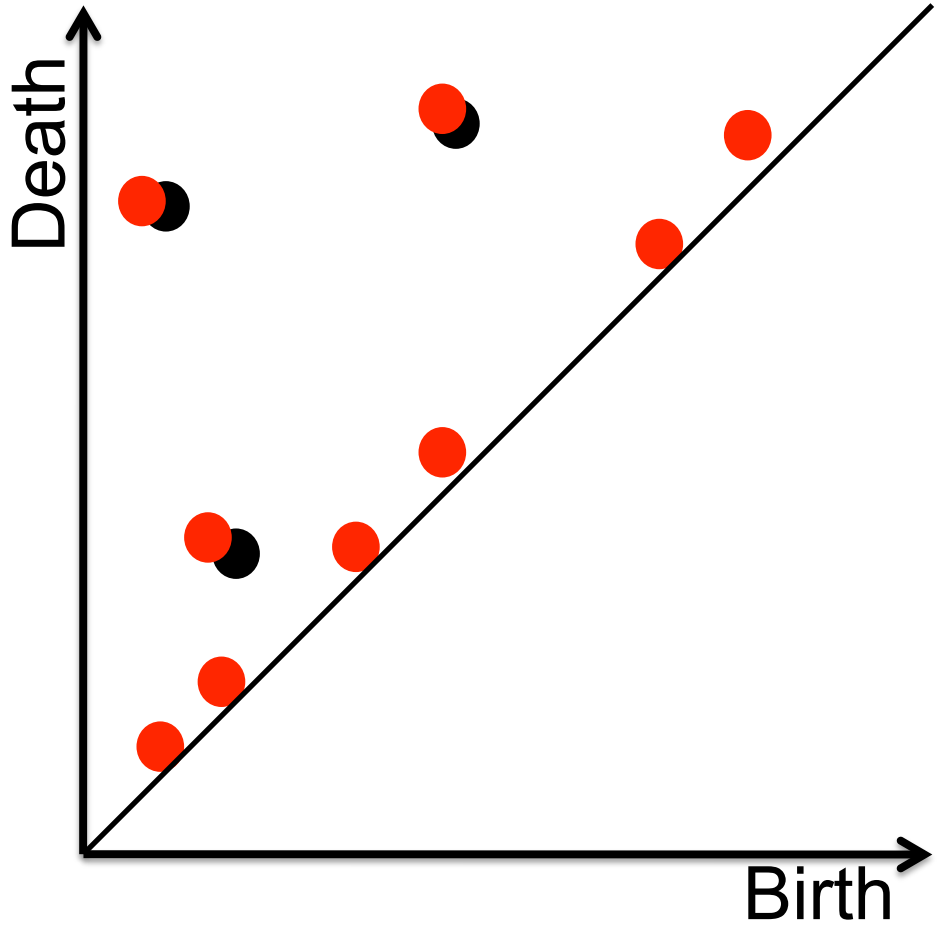


Persistent Homology

1D Pattern



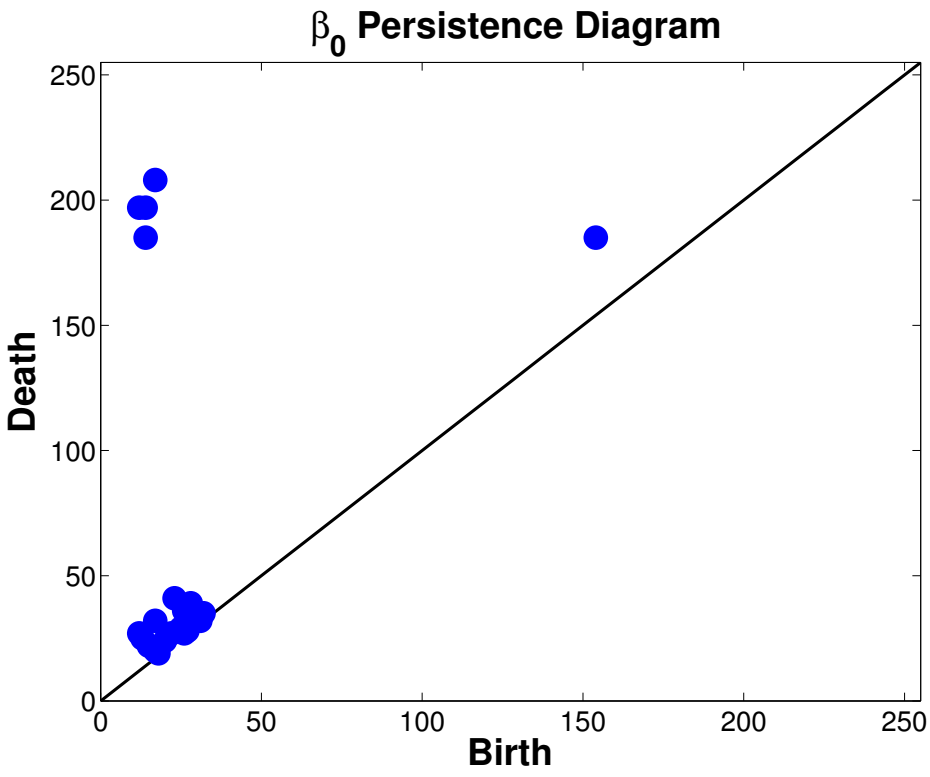
Persistence Diagram

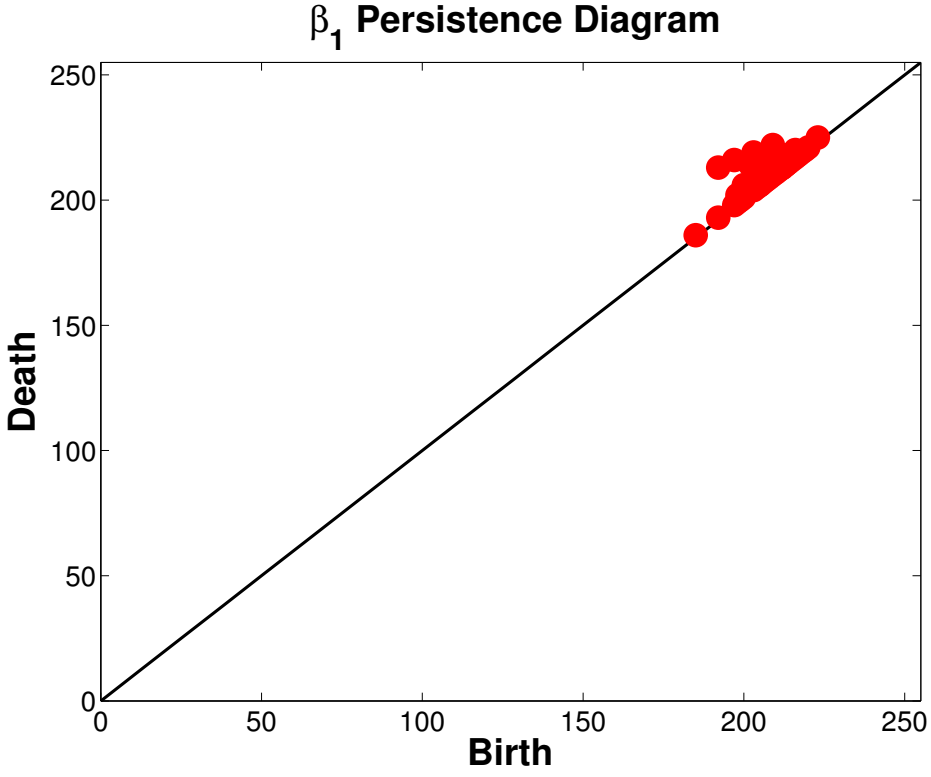


Example Image

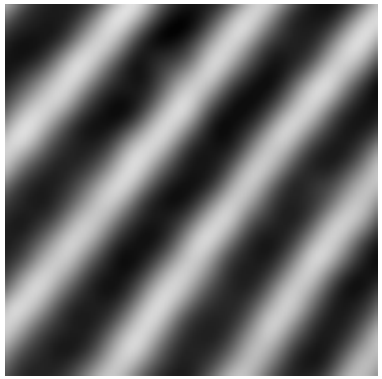
β_0: distinct connected components

β_1: holes

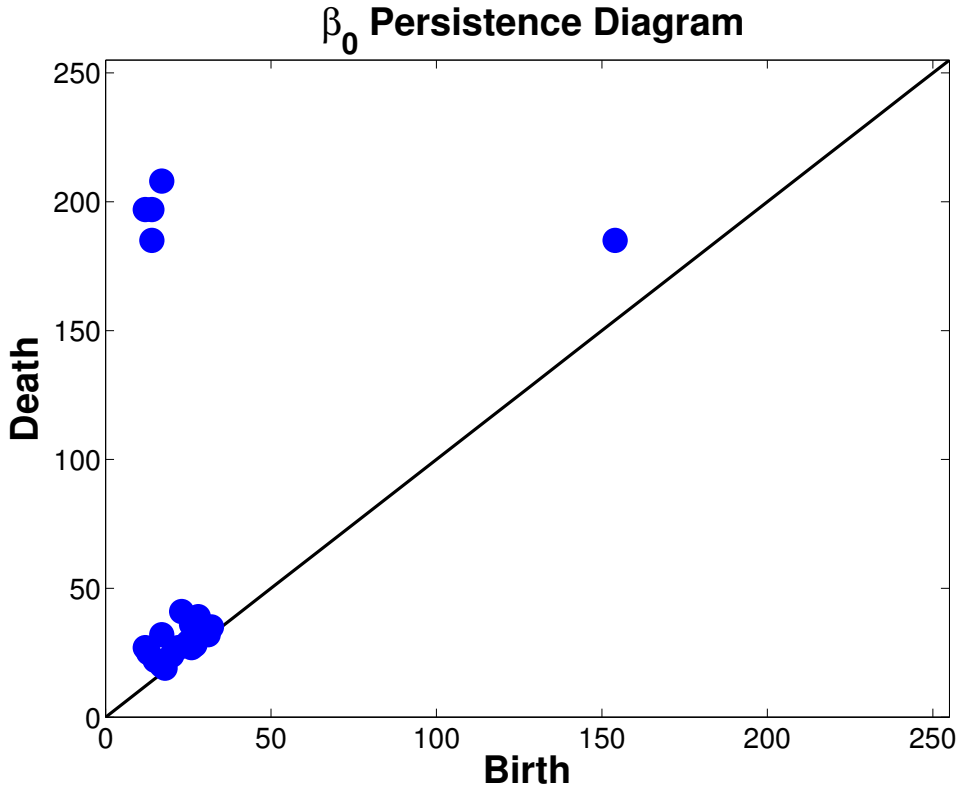


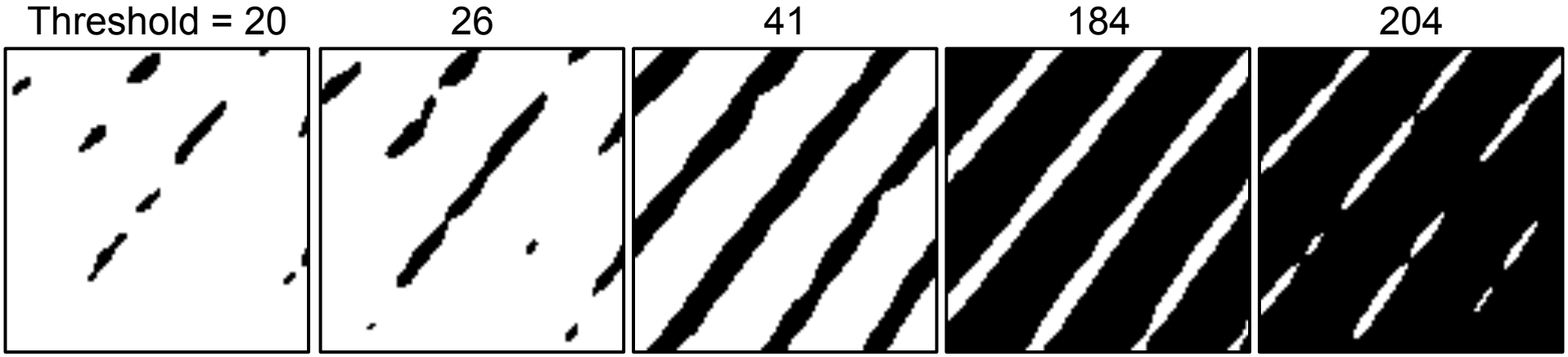


β_0: distinct connected components

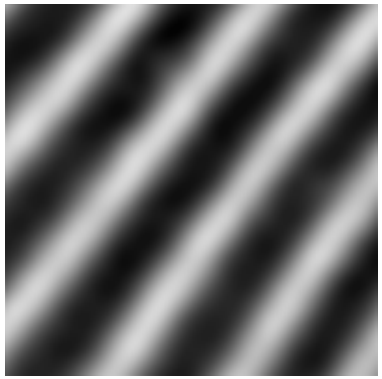


8 bit image
(Pixel Intensity
Range: 0-255)
Low – dark / hot
High – light / cold





β_1: holes



8 bit image
(Pixel Intensity
Range: 0-255)
Low – dark / hot
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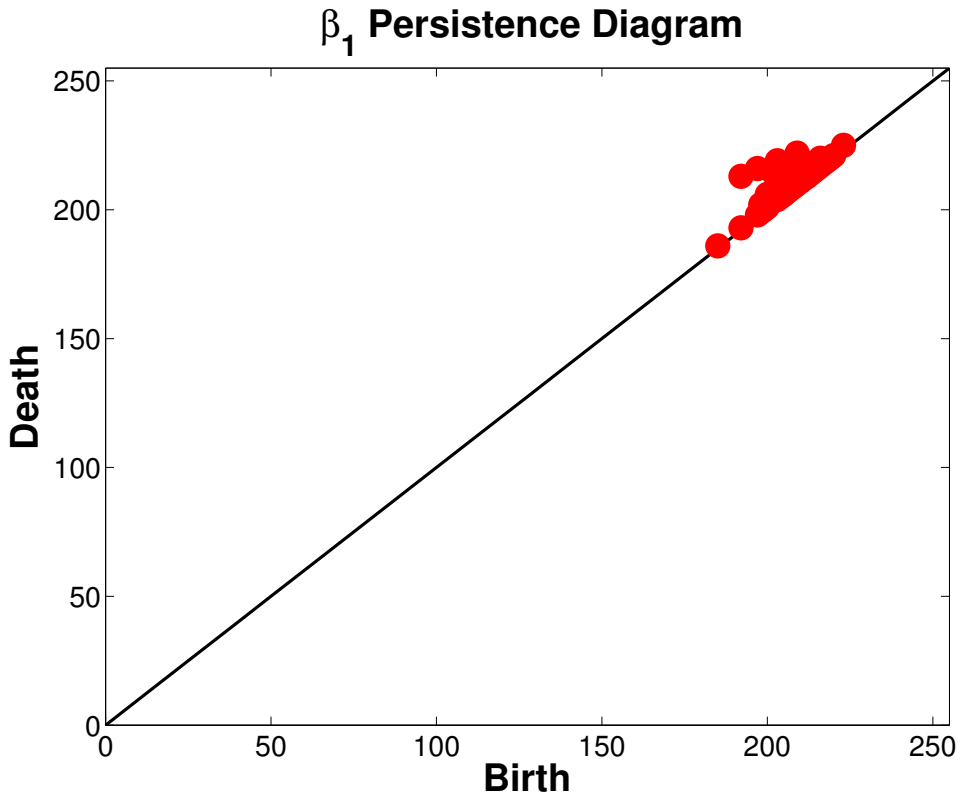
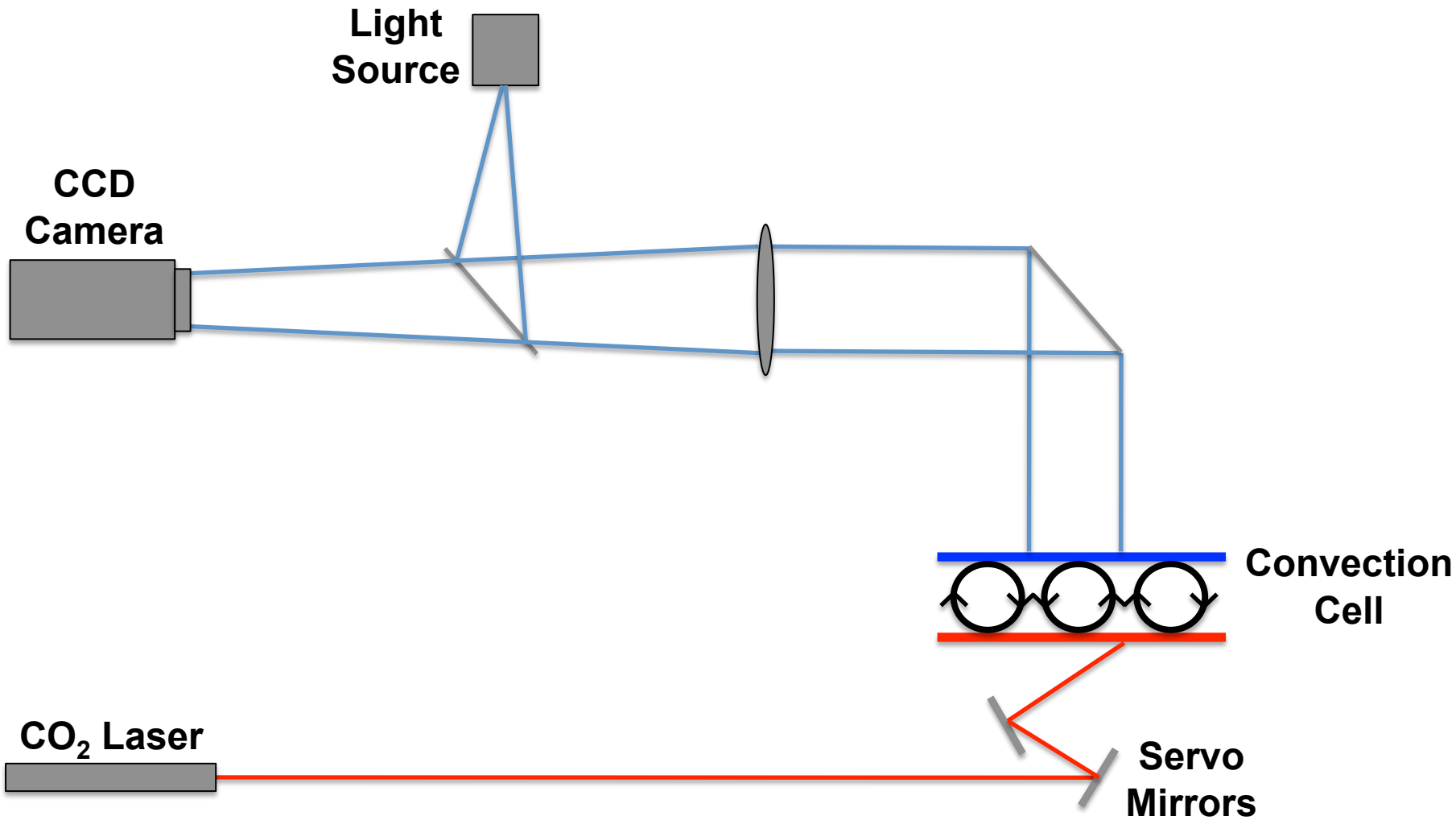


Threshold = 184 194 204 214 219

Experimental Setup



Imposing the Target Pattern

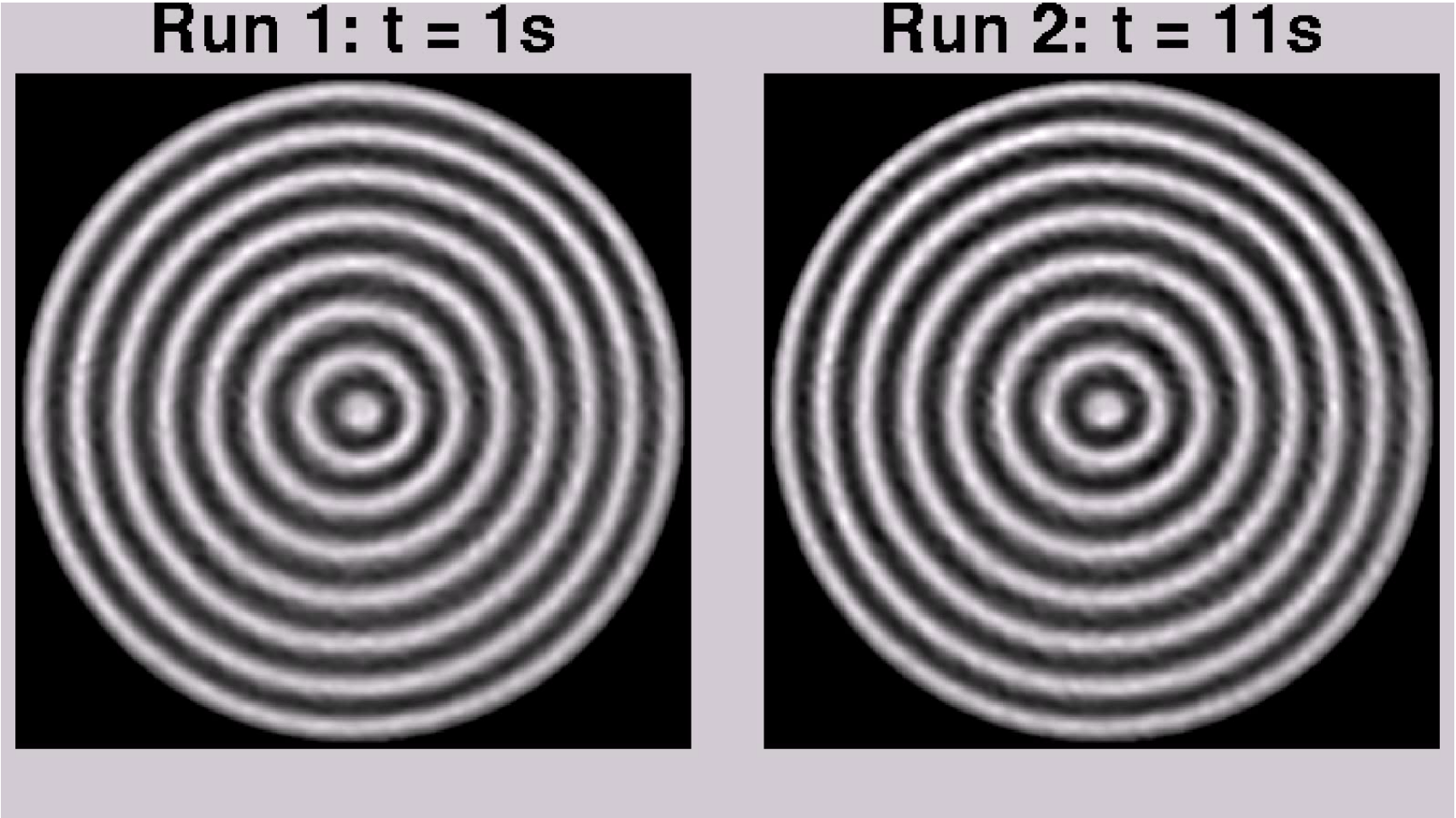
60 x Real Time

Light: hot flow moving upward (out of page)

Dark: cold flow moving downward (into page)

Bright White Flashes: Laser heating the fluid

Experimental Data



Ra ≈ 2250

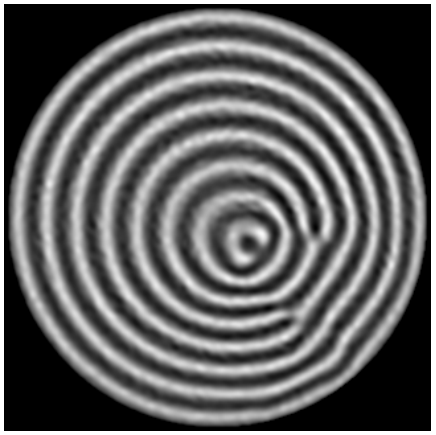
Prandtl Number, $\sigma = 0.84$

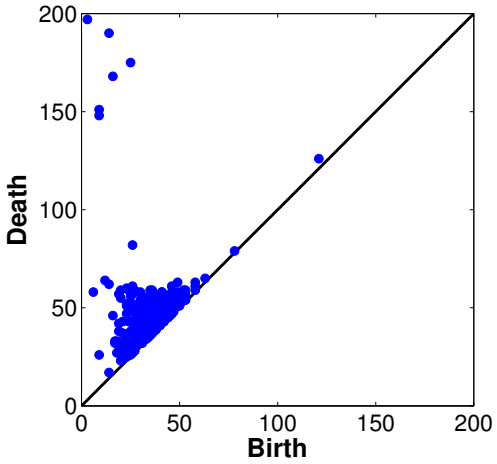
Aspect Ratio, $\Gamma = 19.5$

Vertical Diffusion Time, $t_v \approx 2.6$ s

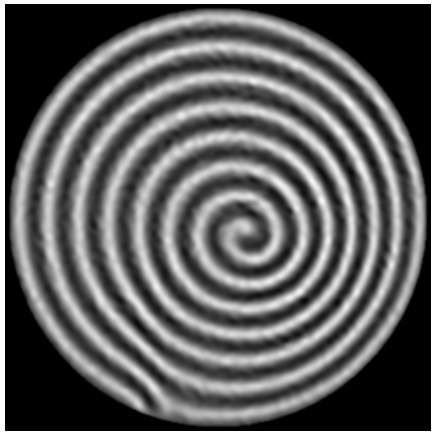
Persistence Diagrams

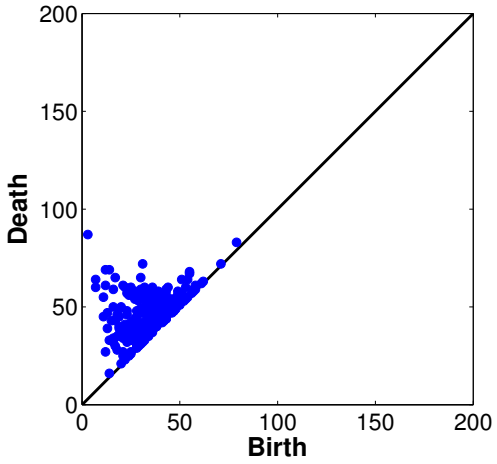
t=194s

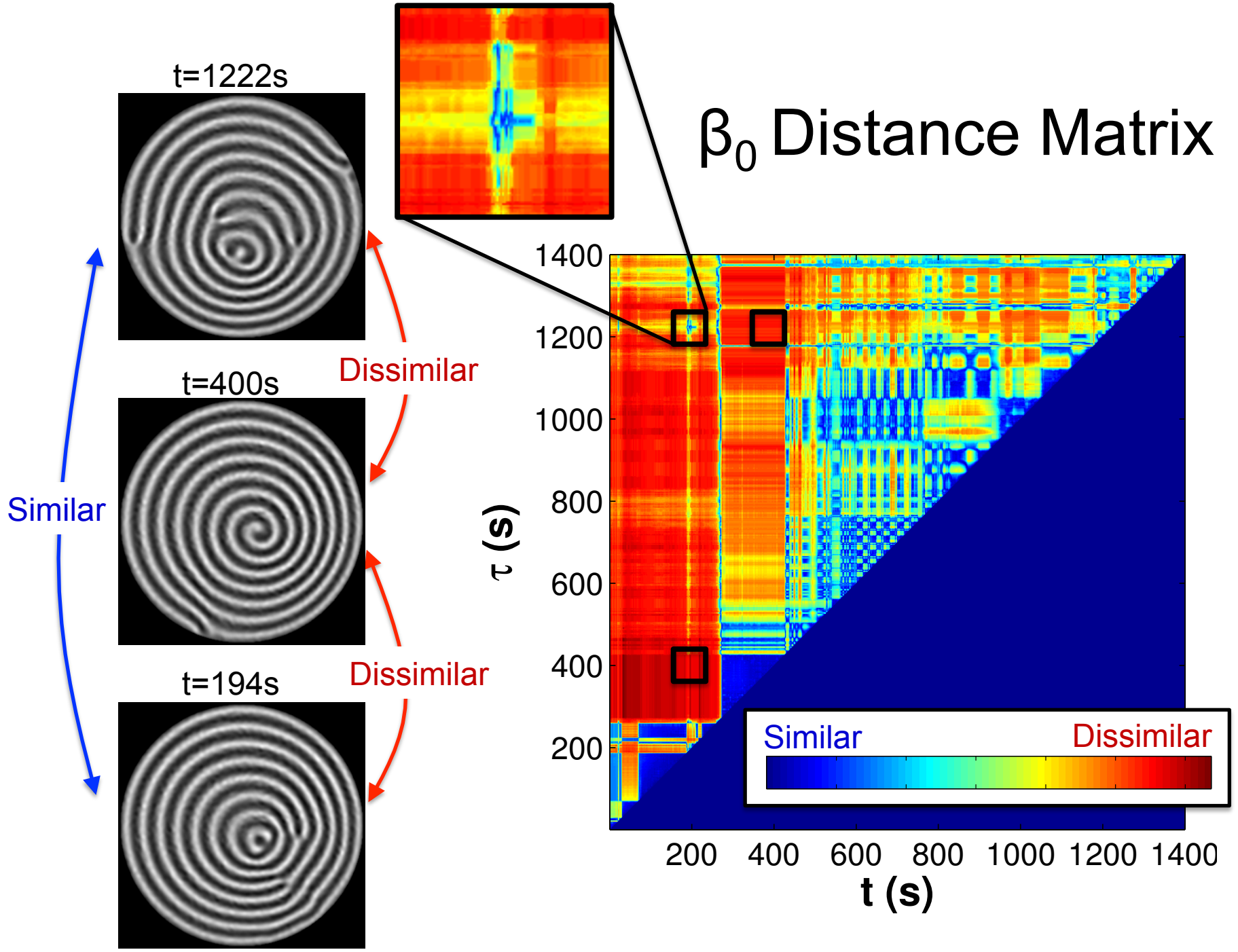


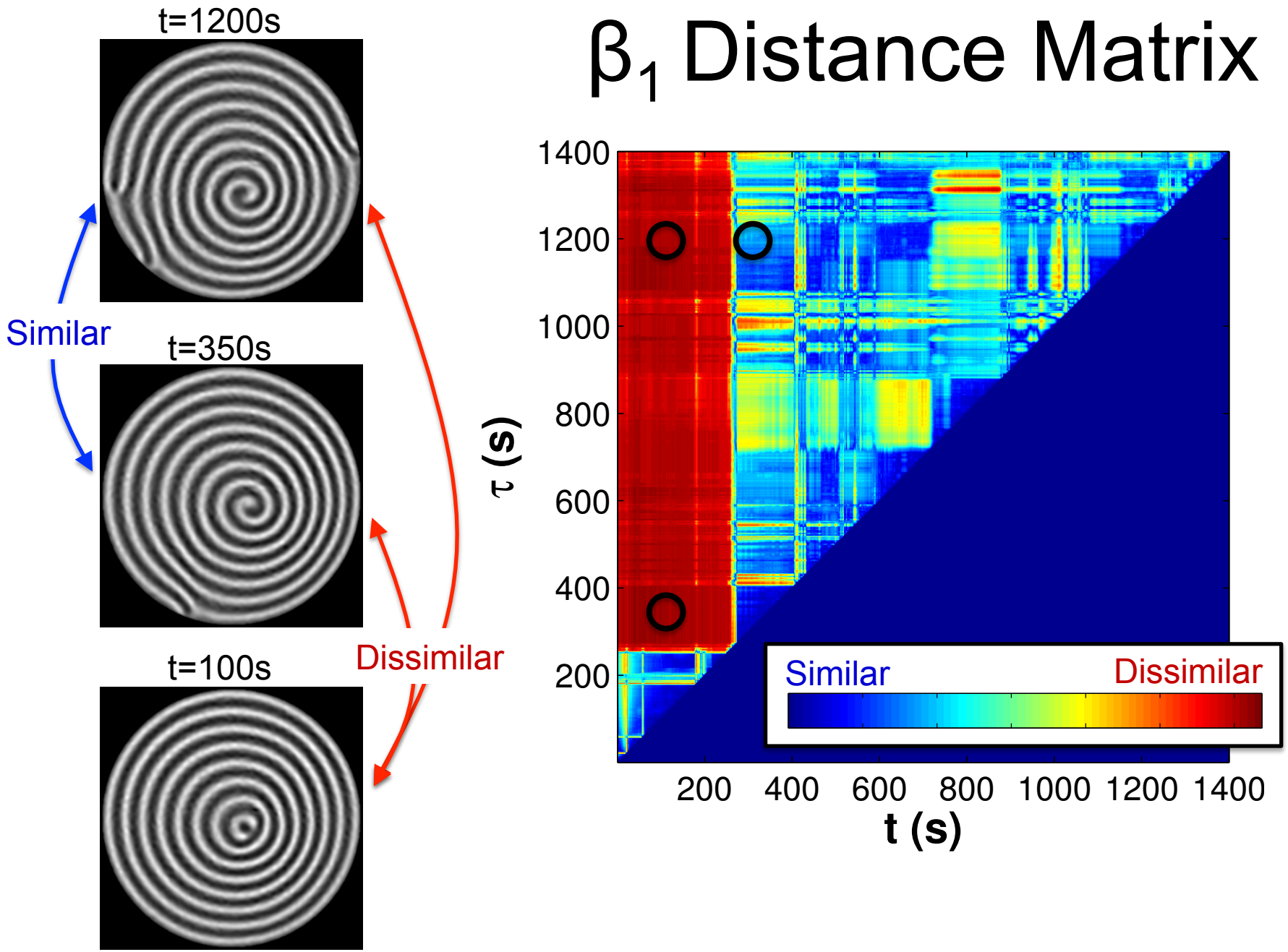


t=400s

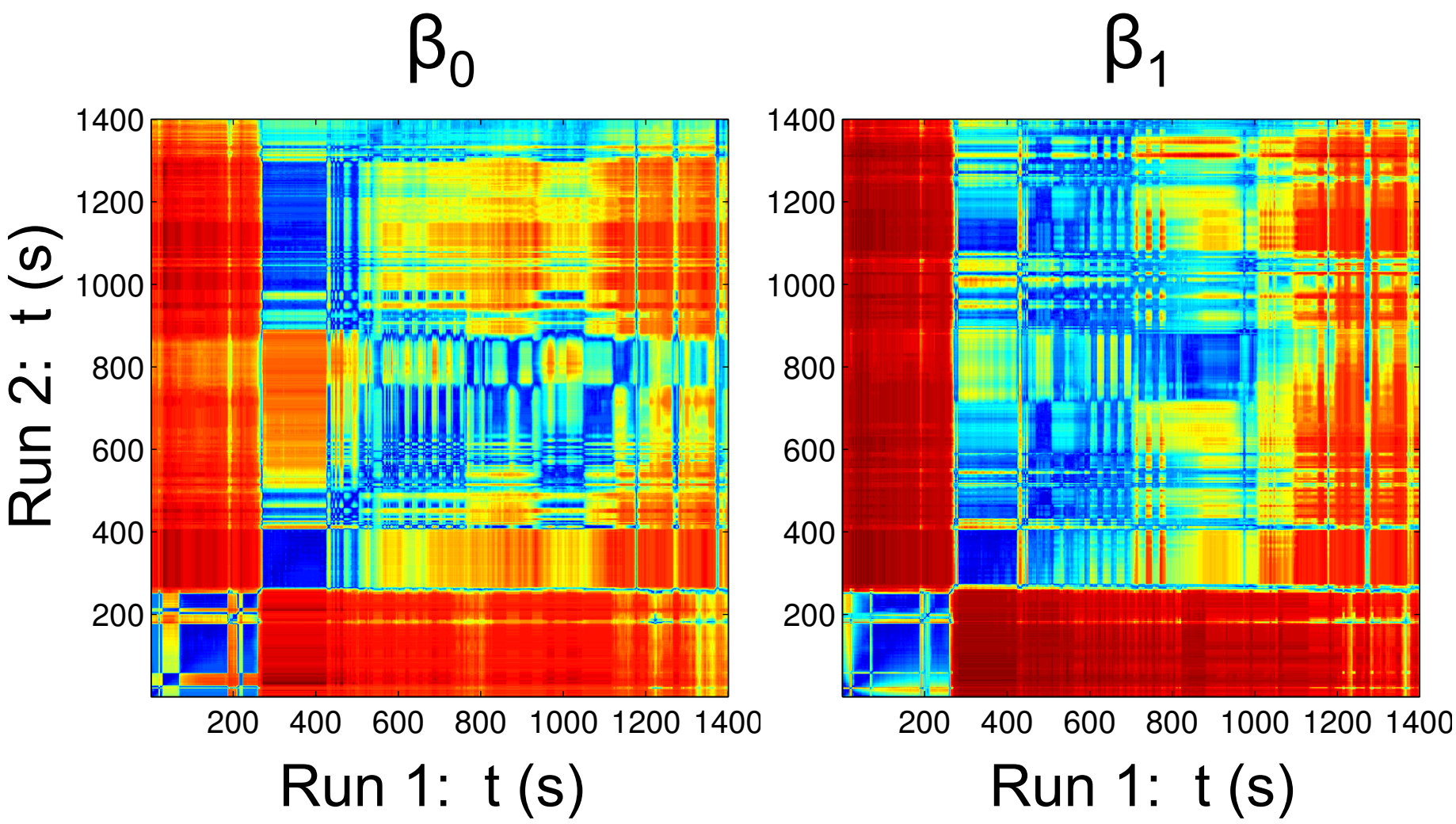





t=1222s
t=400s
t=194s
Similar
Dissimilar
Dissimilar
β0 Distance Matrix
1400
1200
1000
800
600
400
200
τ (s)
200
400
600
800
1000
1200
1400
t (s)
Similar
Dissimilar

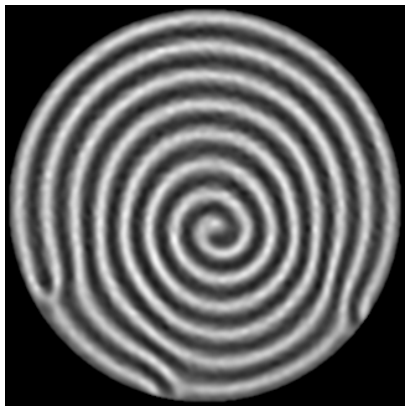

t=1200s
t=350s
t=100s
Similar
Dissimilar
β1 Distance Matrix
τ (s)
t (s)
1400
1200
1000
800
600
400
200
Similar
Dissimilar

Distance Matrix Cross Comparisons

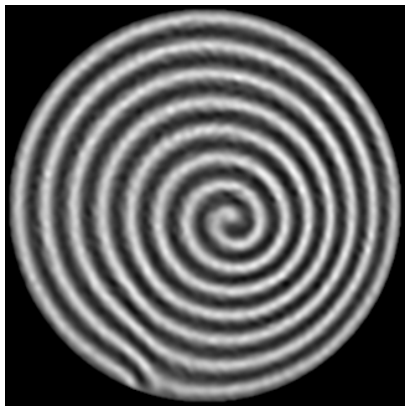


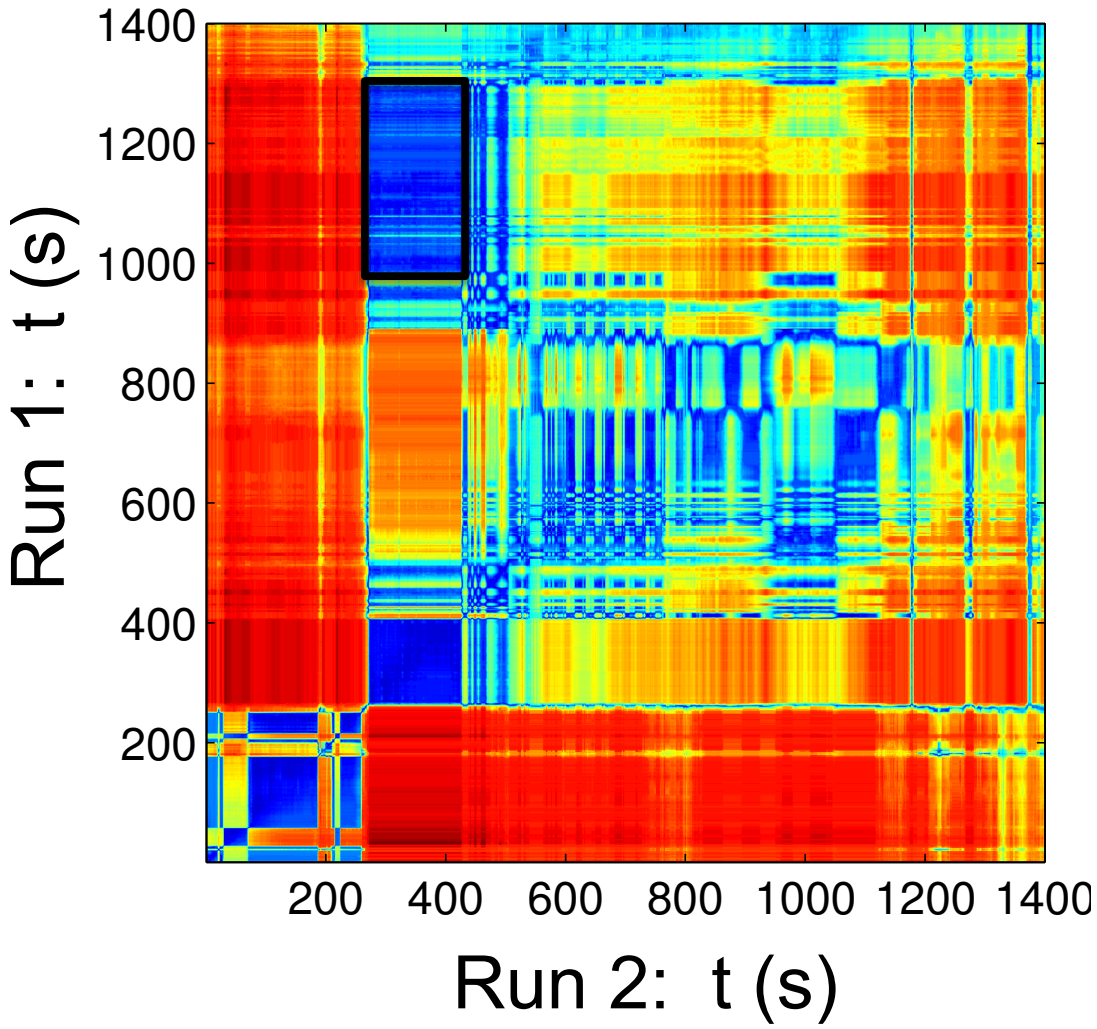
Distance Matrix Cross Comparison

Run 1: t=1100s



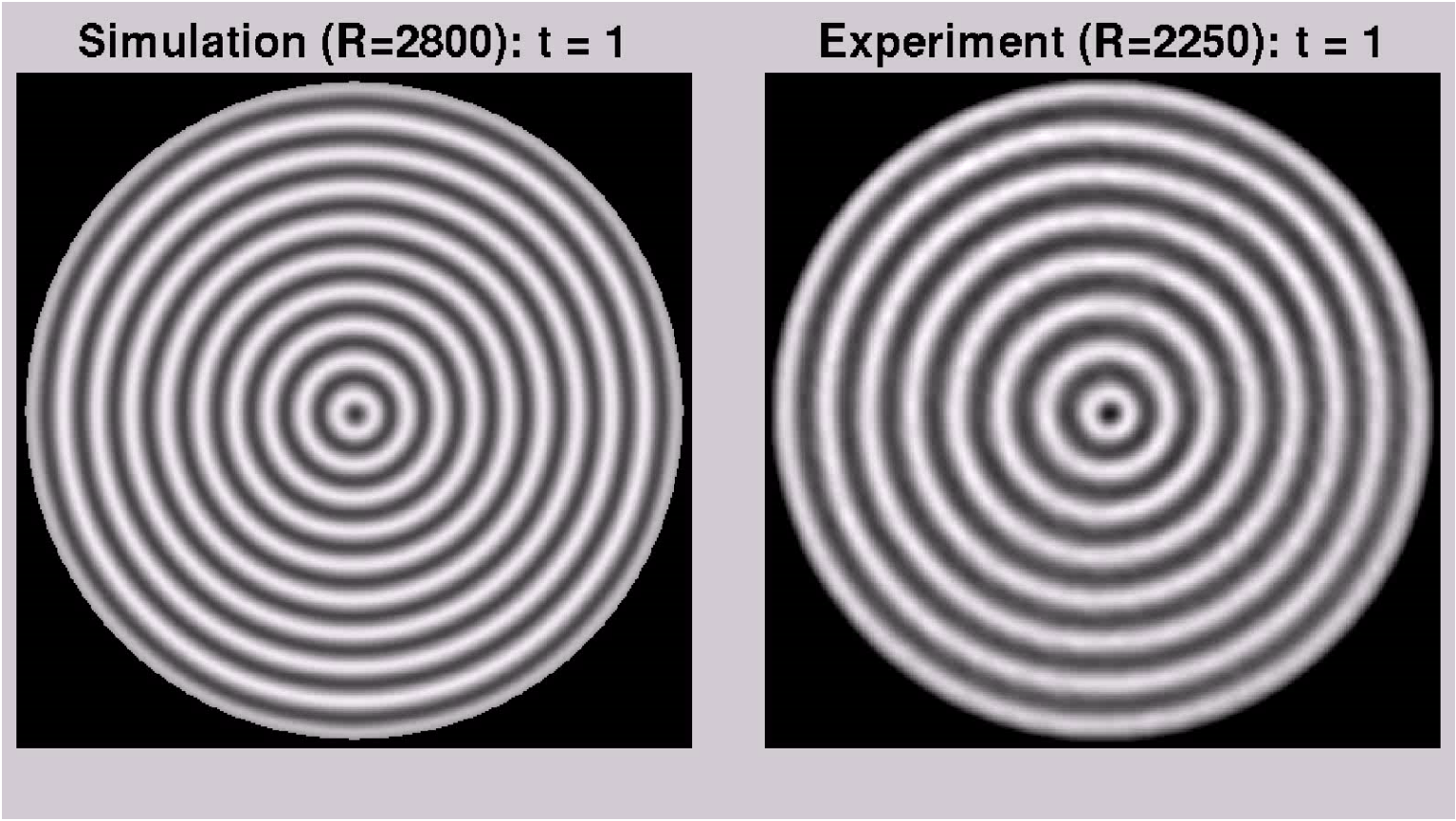
Run 2: t=350s





Future Work

- Use Persistent Homology to directly compare experiment and numerical simulation

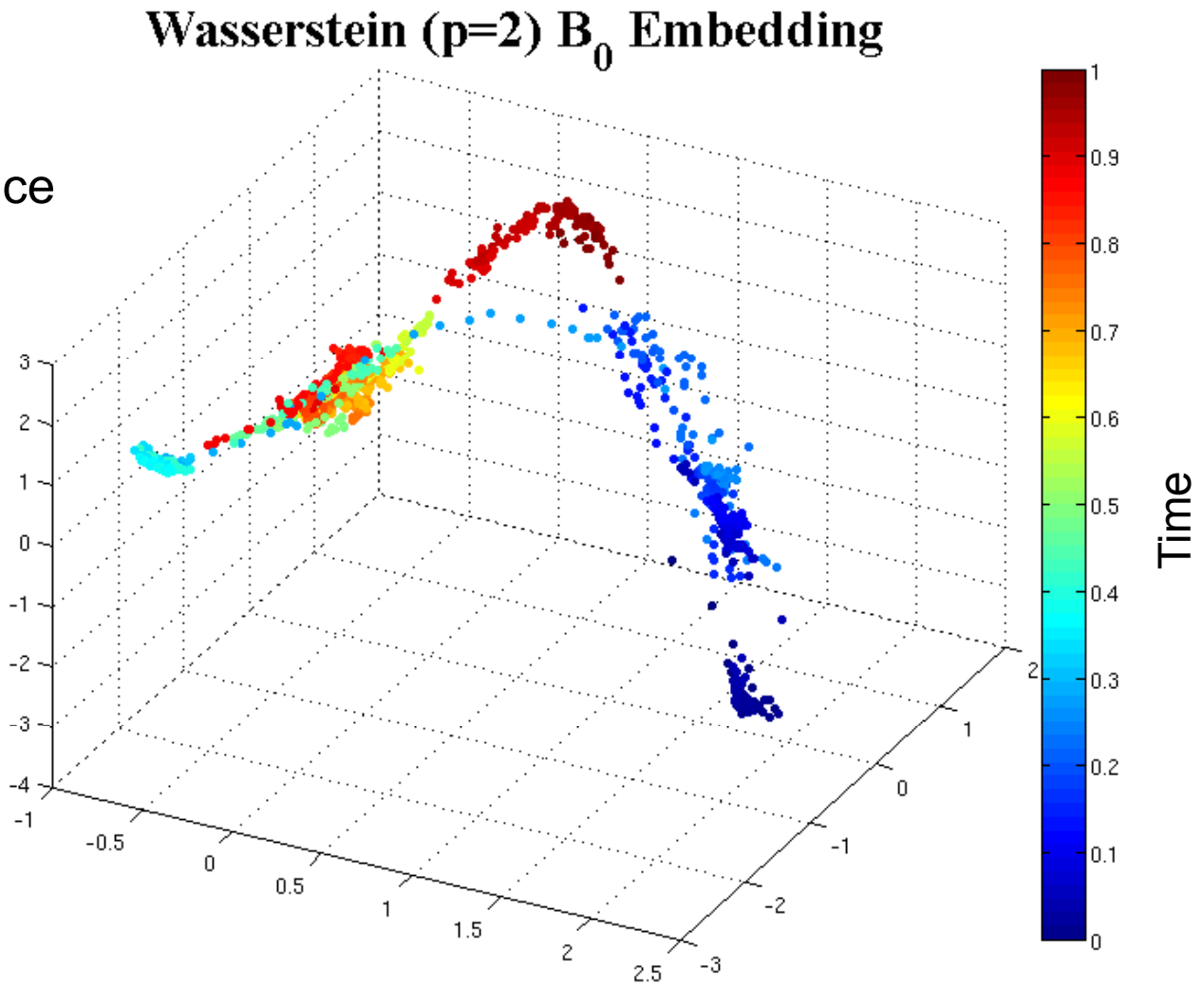


Future Work

- Current distance matrix calculations scale as n^3
  - Two run comparison data took over a week to process on ~200 cores
- New method which scales as n (speed up of about 1,000,000)

Future Work

- Axes: 3 Strongest Eigenvectors from Space of Persistence Diagrams



Conclusions

- New method for characterizing patterns
  - Uses sensitive measures of global topological characteristics
- New method to identify recurrent dynamical behavior
- Quantitatively connect experiment and simulation

Thank you! Questions?