Search for Exact Coherent Structures in a Kolmogorov-Like Flow

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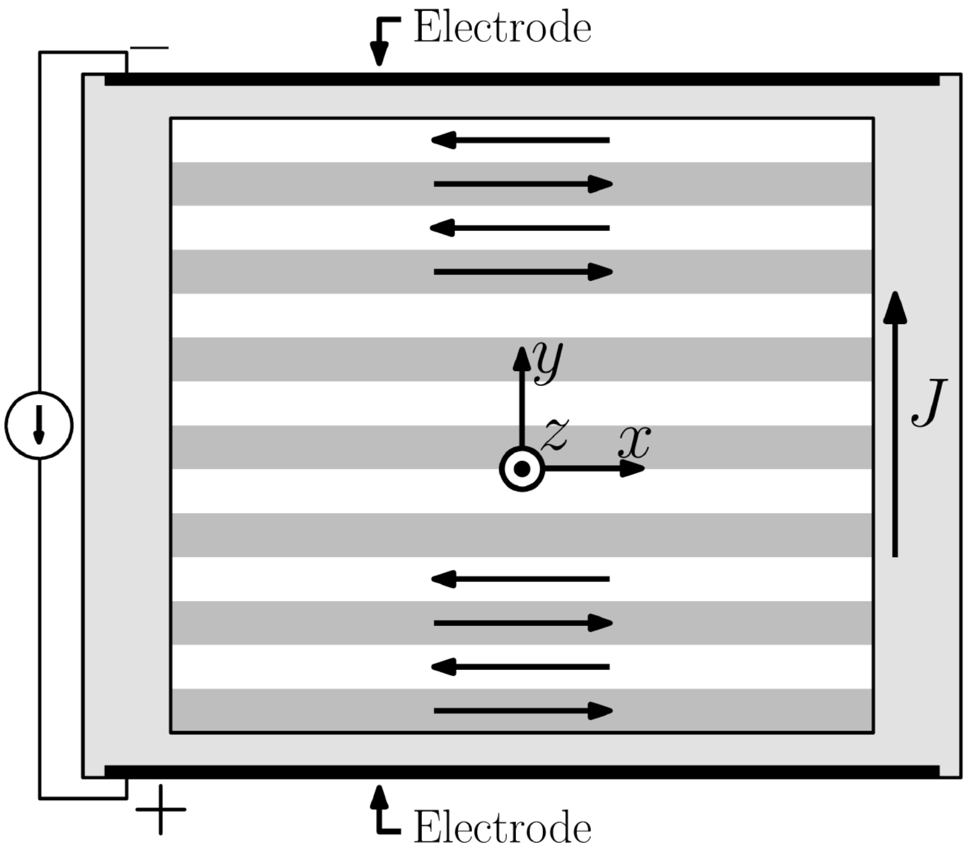
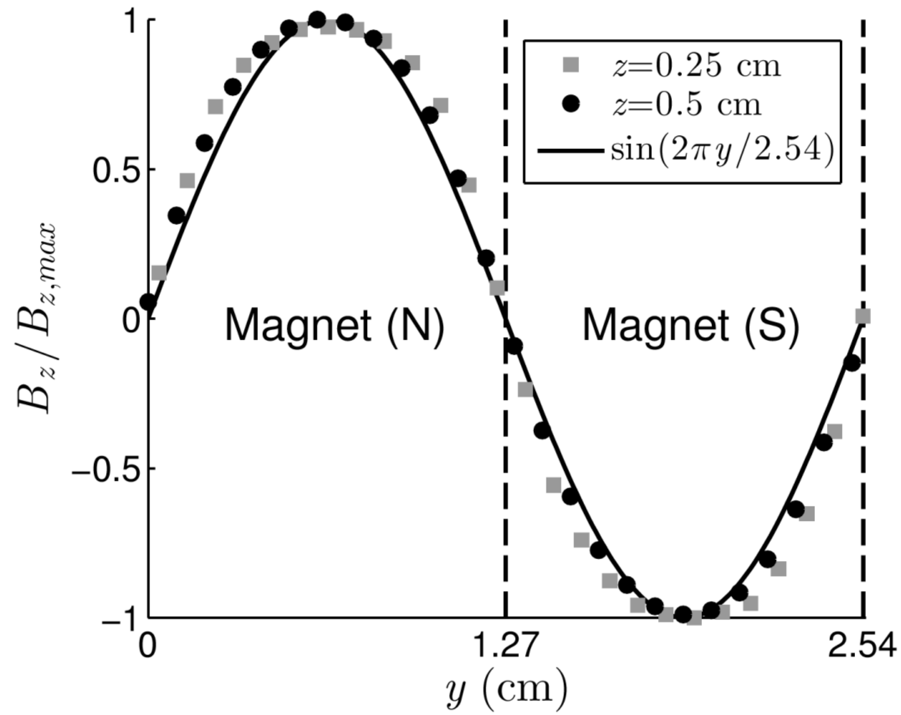
Quasi Two-Dimensional Flows

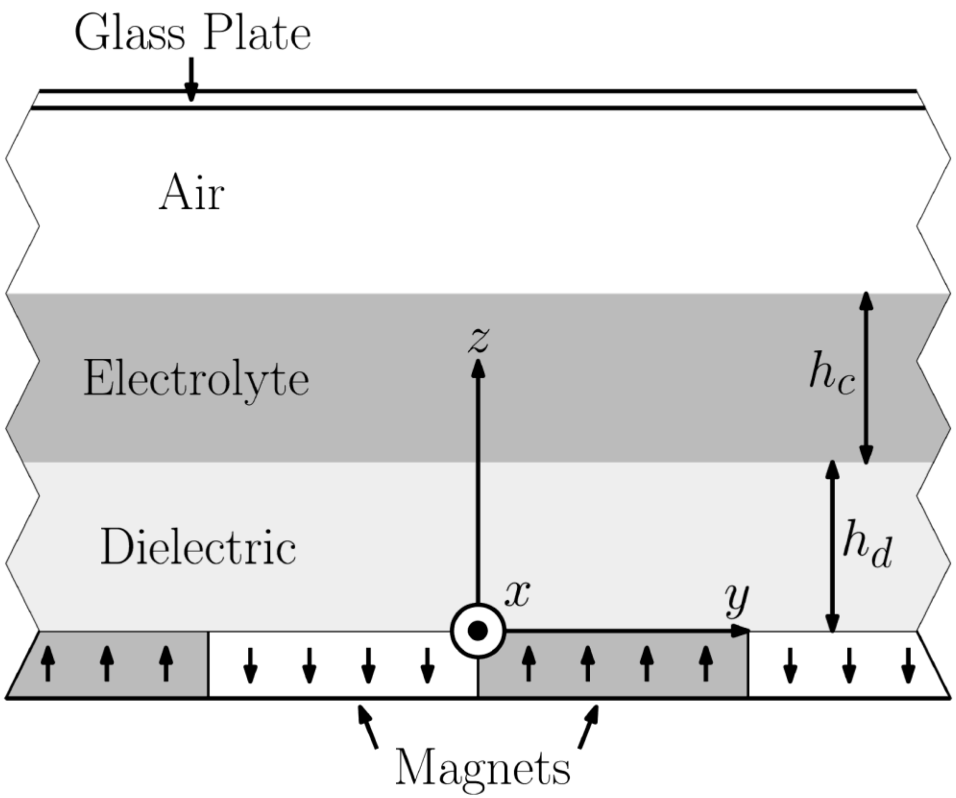
- Flows with ***in-plane*** velocities much larger than ***out-of-plane*** velocities
  - Geometric confinement, rotation, stratification etc.
- Flows in atmosphere, rivers, oceans are often modeled as being Q2D
- Convenient for experimental and theoretical study

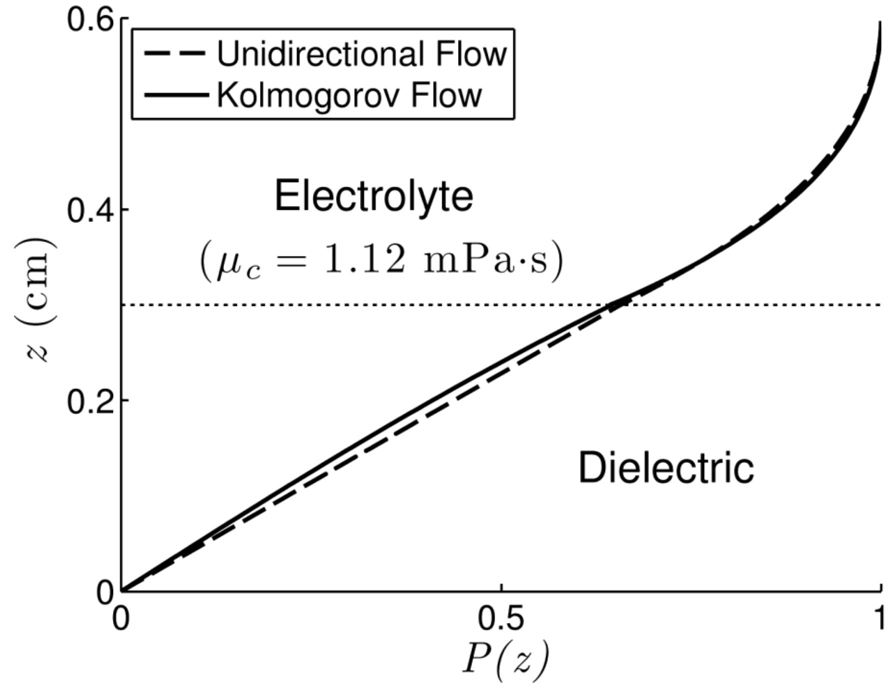
Experimental Setup





Vertical Profile





Depth-Averaged 2D Model

- Governing equations

$$\boxed{\begin{aligned}&\frac{\partial \omega}{\partial t} + \beta U.\nabla \omega = \nu \nabla^2 \omega - \alpha \omega + W \\ &W = A\cos\left(\frac{2\pi}{\lambda} y\right)\end{aligned}}$$

$$\boxed{\nu = 3\,\mathrm{cSt},\ \alpha = 0.07\,\mathrm{s}^{-1},\ \beta = 0.83,\ \lambda = 2.54\,\mathrm{cm}}$$

- Relative importance of dissipation terms

$$\boxed{\frac{\lambda^2 \alpha}{\nu} \approx 15}$$

Symmetries

$$\boxed{\frac{\partial\omega}{\partial t}+U.\nabla\omega=\nu\nabla^2\omega-\alpha\omega+A\cos\left(\frac{2\pi}{\lambda}y\right)}$$

- Inversion/ Rotation by π

  X → -X, Y → -Y
- Half Shift along **Y** + Reflection in **Y-axis**

  Y → Y + λ/2, X → -X
- Translational Invariance along **Y-axis**

  X → X + ε

Computational Details

- Computational Grid Size: 128 x 128
- Spectral Code → Periodic in both **X, Y** directions
- Number of symmetry related copies (isomers)
  - Discrete shifts in transverse direction = 2 * Number of Periods
  - Continuous shifts in longitudinal direction = Number of Grid Points (arbitrary = 128)
  - Inversion symmetry = 2
  - Total Symmetry Operations 2*128*8 = 2048

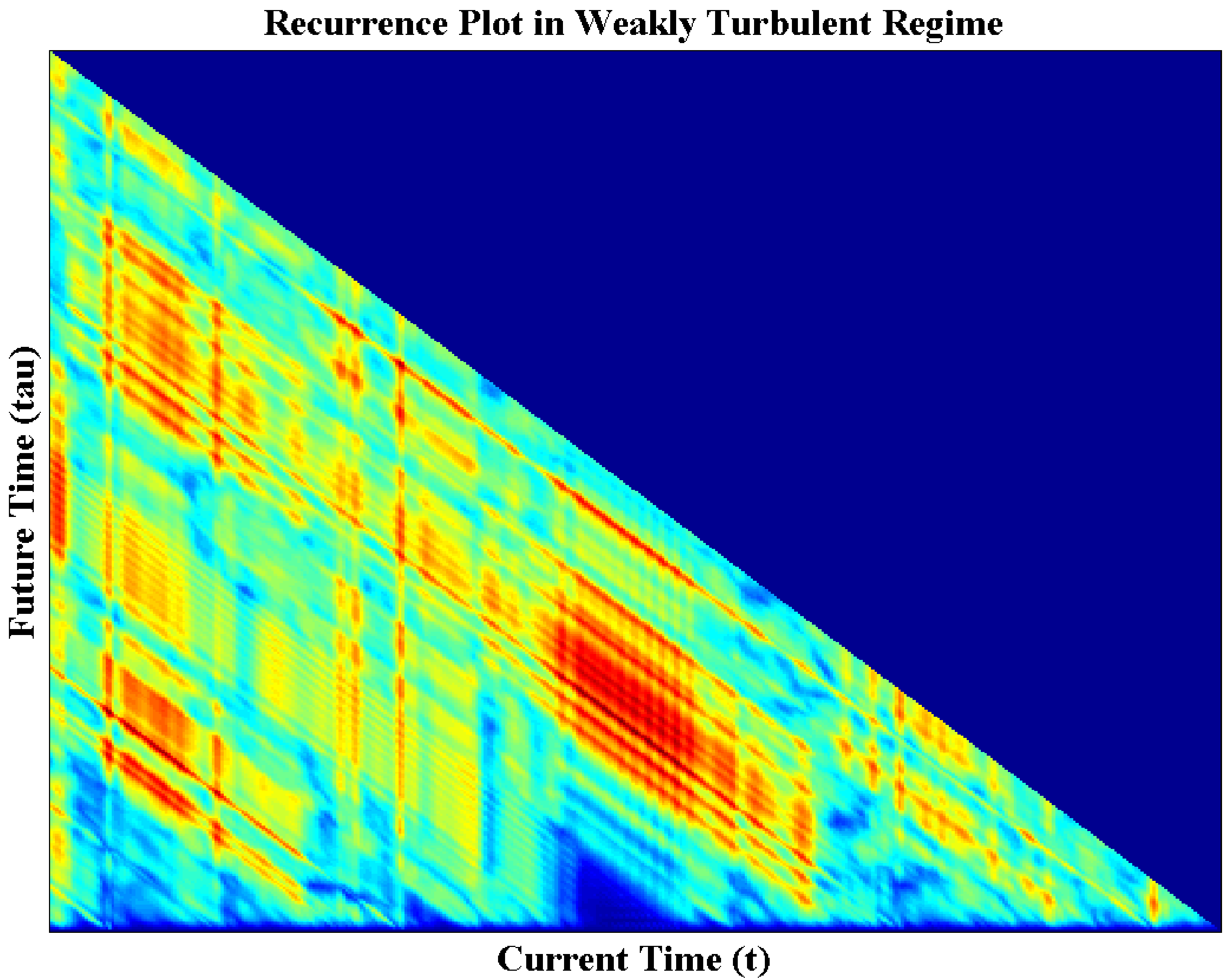
Computational Time

- Each point in recurrence plot corresponds to subtracting the two matrices (128X128), as many times as the number of isomers (1024)
- Typical recurrence plot size → 300x300/2
- Typical time for two 128x128 matrix to be subtracted from each other for 1024 times → 2.5 seconds
- Minimum time for computing recurrences

  →2.5 x 300 x 300/2 $\approx$ 3 hrs
- Usually Takes 6 hours

Optimisation

- Do high frequencies effect the recurrence search?
  - Reduce the 128x128 grid to a 32x32 grid for computing recurrences
  - Reduces computation time by a factor ≈ 10
  - Parallelise the code in MATLAB
- Present computation time for 500x500 is less than 2 hours
- Recurrence analysis for this system is not the bottle neck
- Convergence of solution from initial guesses takes a few hours on average


Recurrence Plot in Weakly Turbulent Regime
Future Time (tau)
Current Time (t)

Some Incoherent Thoughts and Questions!!

- We know that the boundaries play a major role in determining flow dynamics. Do symmetries play role in such cases??
- From a purely theoretical perspective, I believe system size is a very crucial parameter. So, sticking to the doubly periodic case, we should choose the smallest possible cell for dynamics. Forgetting the idea of achieving turbulence, it may help in connecting the various solutions that move in a confined space