Problem 1

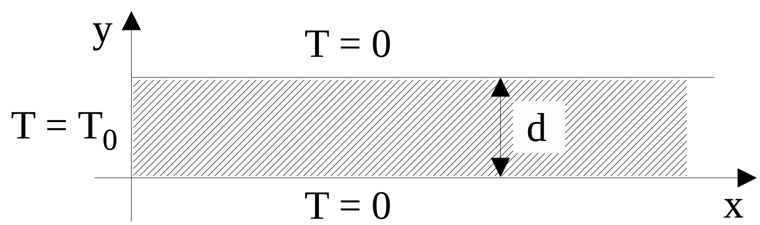
a) Consider the problem of finding the potential in the upper-half plane, if the potential along the x-axis is $\phi(x,0) = V_0$, $|x| < a$, and $\phi(x,0) = 0$, $|x| > a$. Show that this is the same problem as finding the potential due to two line charges, if we exchange the roles of potential and stream function. The solution is

$$\phi = \frac{V_0}{\pi}\left(\arctan\frac{x+a}{y} - \arctan\frac{x-a}{y}\right).$$

b) With the aid of the solution to part a, and the transformation

$$w = \cosh\frac{\pi z}{d} \qquad \text{or} \qquad w = \cosh\frac{\pi z}{2d}$$

(is it conformal?) find the temperature distribution in the semi-infinite metal plate whose cross-section is shown below, if the three faces are maintained at the indicated constant temperatures. Note that $\nabla^2 T = 0$ inside the metal.



Problem 2

Using conformal mapping technique find the potential between two infinite planes intersecting along the z-axis and making an angle α. The planes are charged to the potential $\phi_1 = a\rho^{\pi/2\alpha}$ and $\phi_2 = b\rho^{\pi/2\alpha}$, where ρ is the distance from the z-axis and a, b are constants (see figure). (Hint: try mapping the region into the first quadrant of the complex plane.)
a) Write down the result in polar coordinates for arbitrary α.
b) Write down the result in Cartesian coordinates for $\alpha = \pi/4$.