mathematical methods - week 12

Mehr Gruppenpest

Georgia Tech PHYS-6124

Homework HW #12 due Thursday, November 13, 2014

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort

Exercise 12.1 *Three masses on a loop* (1) 1 point; (2) 6 points; (3) 1 point; (4) 2 points

Total of 10 points = 100 % score.

2014-11-04 Predrag Lecture 22 Normal modes

I'm enjoying reading Mathews and Walker [12.1] Chap. 16 *Introduction to groups*. You can download it from here. Goldbart writes that the book is "based on lectures by Richard Feynman at Cornell University." Very clever. In this lecture I work through the example of fig. 16.2: it is very cute, you get explicit eigenmodes from group theory alone. The main message is that if you think things through first, you never have to go through using explicit form of representation matrices - thinking in terms of invariants, like characters, will get you there much faster.

2014-11-06 Predrag Lecture 23 Continuous groups: unitary and orthogonal

This lecture is not taken from any particular book, it's about basic ideas of how one goes from finite groups to the continuous ones that any physicist should know. We have worked one example out earlier, in week 7 *Discrete Fourier transform*. It gets you to the continuous Fourier transform as a representation of $\mathrm{U}(1) \simeq \mathrm{SO}(2)$, but from then on this way of thinking about continuous symmetries gets to be increasingly awkward. So we need a fresh restart; that is afforded by matrix groups, and in particular the unitary group $\mathrm{U}(n) = \mathrm{U}(1) \otimes \mathrm{SU}(n)$, which contains all other compact groups, finite or continuous, as subgroups.

The main idea in a way comes from discrete groups: the whole cyclic group C_N is generated by the powers of the smallest rotation by $2\pi/N$, and in the $N \to \infty$ limit one only needs to understand the algebra of T_ℓ, generators of infinitesimal transformations, $D(\theta) = 1 + i \sum_\ell \theta_\ell T_\ell$.

These thoughts are spread over chapters of my book [12.2] *Group Theory - Birdtracks, Lie's, and Exceptional Groups* that you can steal from my website, but the book itself is too sophisticated for this course. If you ever want to learn some group theory in depth, you'll have to petition the School to offer it - last time the course was offered was in 2008.

References

[12.1] J. Mathews and R. L. Walker, *Mathematical Methods of Physics* (Addison-Wesley, Reading, MA, 1973).

[12.2] P. Cvitanović, *Group Theory - Birdtracks, Lie's, and Exceptional Groups* (Princeton Univ. Press, Princeton, NJ, 2008), `birdtracks.eu`.

Exercises

12.1. **Three masses on a loop.** Three identical masses, connected by three identical springs, are constrained to move on a circle hoop as shown in figure 12.1. (1) Find all symmetries

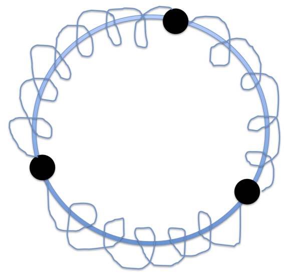

Figure 12.1: Three identical masses are constrained to move on a hoop, connected by three identical springs such that the system wraps completely around the hoop. Find all symmetries of the equations of motion.

of the equations of motion. (2) Find the normal modes using group-theoretic decompositions to irreps and character orthonormality. (3) How many eigenvalues are there, in all? (4) Interpret the eigenvalues and eigenvectors from a group-theoretic, symmetry point of view. (Exercise 2.1 revisited.)