mathematical methods - week 15

Bayesian statistics

Georgia Tech PHYS-6124

Homework HW #15 due Thursday, December 4, 2014

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort

Exercise 15.1 *Bayesian statistics*, by Sara A. Solla 10 points

Total of 10 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

2014-11-25 Predrag Lecture 28 Bayesian statistics

Sara A. Solla, Northwestern University

Natural sciences aim at abstracting general principles from the observation of natural phenomena. Such observations are always affected by instrumental restrictions and limited measurement time. The available information is thus imperfect and to some extent unreliable; scientists in general and physicists in particular thus have to face the task of extracting valid inferences from noisy and incomplete data. Bayesian probability theory provides a systematic framework for quantitative reasoning in the face of such uncertainty.

In this lecture we will focus on the problem of inferring a probabilistic relationship between a dependent and an independent variable. We will review the concepts of joint and conditional probability distributions, and justify the commonly adopted Gaussian assumption on the basis of maximal entropy arguments. We will state Bayes' theorem and discuss its application to the problem of integrating prior knowledge about the variables of interest with the information provided by the data in order to optimally update our knowledge about these variables. We will introduce and discuss Maximum Likelihood (ML) and Maximum A Posteriori (MAP) for optimal inference. These methods provide a solution to the problem of specifying optimal values for the parameters in a model for the relationship between independent and dependent variables. We will discuss the general formulation of this framework, and demonstrate that it validates the method of minimizing the sum-of-squared-errors in the case of Gaussian distributions.

- A quick but superficial read: Matthew R. Francis, *So what's all the fuss about Bayesian statistics?*
- Reading: Lyons [15.1], *Bayes and Frequentism: a particle physicist's perspective* (click here)

References

[15.1] L. Lyons, Bayes and Frequentism: a particle physicist's perspective, Contemporary Physics **54**, 1 (2013).

Exercises

15.1. **Bayes.** *Bayesian statistics*.

Sara A. Solla