Taylor Couette Simulations

By
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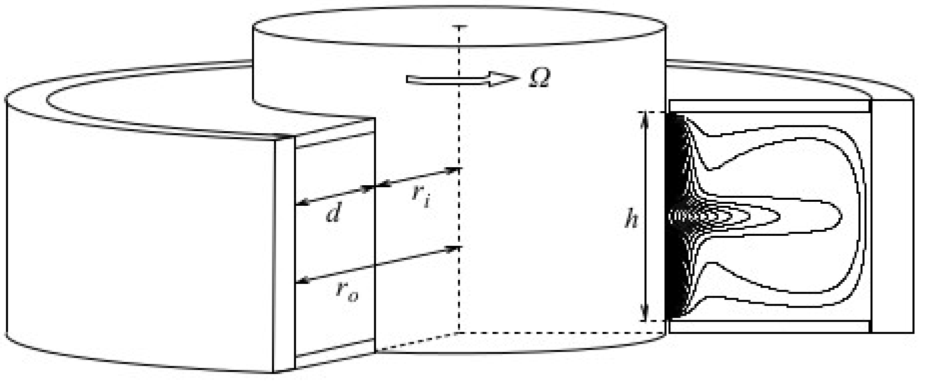
Numerical Scheme

- Spectral Solver
- Parameters (Experimental Values)
  - Viscosity : ν=1.39 (mm^2/sec) @ 21.5 Celsius
  - Height of Cylinders : h=40mm
  - Inner Radii : $r_i^* = 72.4 \ \ mm$
  - Outer Radii : $r_o^* = 80 \ \ mm$
  - Gap Length : $d = r_o^* - r_i^* = 7.6 \ \ mm$
- Non-dimensionize using a length and time scale
  - Length Scale : d
  - Time Scale : $\frac{d^2}{\nu}$

Numerical Scheme Continued...

- Geometry Parameters
  - Aspect Ratio: $\Gamma = \frac{h}{d}$
  - Radius Ratio: $\eta = \frac{r_i^*}{r_o^*}$
- Renolds Number
  - $\mathrm{Re}_i = \frac{\Omega_i r_i^* d}{\nu}$
  - $\mathrm{Re}_o = \frac{\Omega_o r_o^* d}{\nu}$



- Flow governed by the incompressible Navier-Stokes equations. In cylindrical coordinates (r,θ,z) the non-dimensional Navier-Stokes equations are

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\nabla p + \nabla^2 \vec{v}$$

$$\nabla \cdot \vec{v} = 0$$

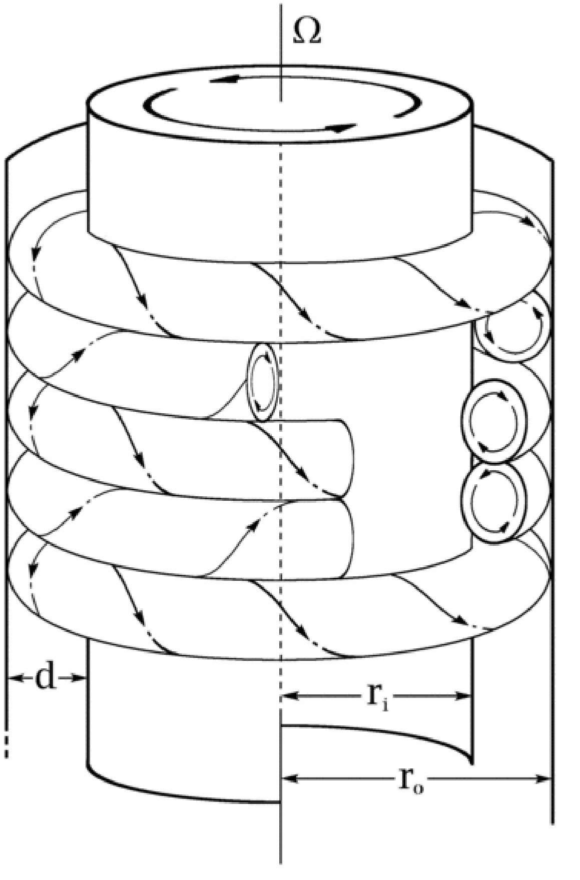
- $\vec{v} = (u, v, w)$ and p are the non-dimensional velocity vector and pressure

Methods

- Navier Stokes Equations are solved using a second order time splitting method
- Spatial Discretization
  - Chebyshev Collocation in r & z
    - $r_i = \frac{1}{2}\left(\cos\left(\frac{i\pi}{N_r}\right) + r_i + r_o\right) \qquad i = 0, \ldots, N_r$
    - $z_j = \frac{\Gamma}{2}\cos\left(\frac{j\pi}{N_z}\right) \qquad j = 0, \ldots, N_z$
  - Galerkin-Fourier Expansion in θ
  - Note: Nr and Nz are the dimensions of the radial and axial Chebyshev points respectively
- $F(r,\theta,z) = \sum_{l=0}^{L} \sum_{n=0}^{N} \sum_{m=0}^{M} a_{l,m,n} T_n(x) T_m(y) e^{im\theta}$
- With $x = 2r - r_i - r_o$ and $y = \frac{2z}{\Gamma}$. The three velocity components and pressure are represented by f=real(F).

Methods Continued...

- The Helmhotz and Poisson Equations for each fourier mode are solved efficiently using a complete diagonalization of the operators in the radial and axial directions



Boundary Conditions

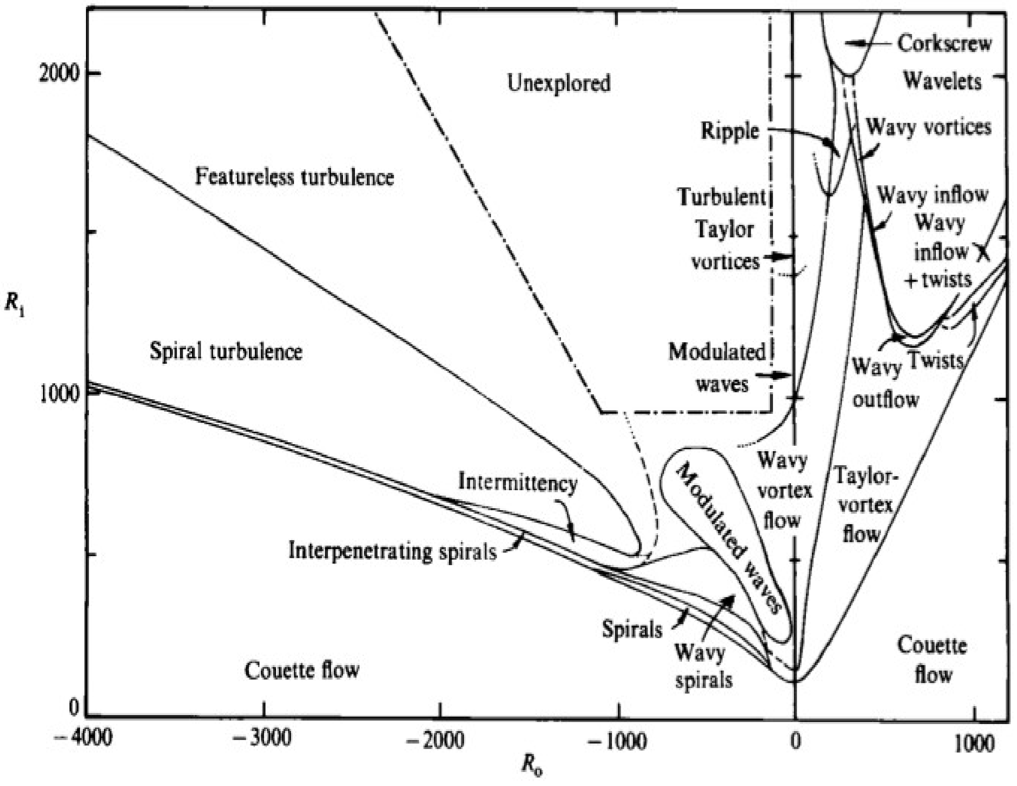
- Implemented the GT experiment boundary conditions
- The outer cylinder is connected to the end plates.
- This corresponds to the following boundary conditions:
  - $v(r_i,\theta,z,t)=(0,\mathrm{Re}_i,0)$
  - $v(r_o,\theta,z,t)=(0,\mathrm{Re}_o,0)$
  - $v(r,\theta,\pm\frac{\Gamma}{2},t)=(0,\mathrm{Re}_o,0)$
- There is a discontinuity where the inner cylinder meets the end plates. For an accurate use of spectral techniques, the discontinuity is implemented using the form below

$$v\left(r,\theta,\pm\frac{\Gamma}{2},t\right)=\mathrm{Re}_i\,e^{\frac{r_i-r}{\epsilon}}+\mathrm{Re}_o\frac{(r-r_i)}{d}$$

- Note: Epsilon is a parameter that mimics the small physical gap between the inner cylinder and plate boundary. It is ε=0.005 in the program.

Comparison between Simulations & Experiments

- Verification was done using different values for the Reynolds number. The numbers were predetermined by Daniel.
- Interesting Case: Rei=870 & Reo=0.
  - Description given by Daniel: “4 Taylor rolls with spiral-like defect”
- Less interesting case: Rei=746 & Reo=550
  - Description: “4 Taylor rolls”



Simulation Video

- See video

Current & Future Projects

- Double checking the correctness of the Recurrence Plots
- Implement GMRES into FORTRAN
- Begin finding periodic orbits once GMRES is finished.
- Impliment an algorithm to find relative periodic orbits.

Special Thanks

- Marc Avila – Author of original FORTRAN code
- Daniel Borrero – Experimental values and comparison between Simulation and Experiment.
- Dr. Grigoriev – Greatly appreciated advise on overcoming constant hurdles within the program.