(C) Project Description:

a. List of Participants

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The key to the school and departmental abbreviations used throughout the proposal:

GT	Georgia Institute of Technology
COS	College of Sciences
COE	College of Engineering
Bio	School of Biology
AE	School of Aerospace Engineering
CE	School of Civil & Environmental Engineering
ECE	School of Electrical & Computer Engineering
ME	School of Mechanical Engineering
Chem	School of Chemistry and Biochemistry
Math	School of Mathematics
Phys	School of Physics

b. Vision, Goals and Thematic Basis

The impact of nonlinear science across the broad spectrum of disciplines is easily documented, and is likely to grow for the coming generation of scientists. In its 2001 report "Physics in a New Era: an Overview" [1], the National Research Council identifies six "grand challenges", among them "understanding complex systems", "applying physics to biology", and "creating new materials", fields where there are no boundaries between physics, mathematics, engineering and life sciences. Within the American Physical Society, the recently created Topical Group on Statistical and Nonlinear Physics is already the largest topical group, while the Division of Biological Physics is one of the fastest growing divisions.

In order to prepare young researchers for these challenges, we propose to move away from the traditional structure of graduate programs and offer a training in the tools and methods of nonlinear science that bridges departmental and school boundaries, national and international research. We envision the **IGERT: School in Nonlinearity & Complexity**(IGERT SINC), fostering

- **Cross-cutting research initiatives, with nonlinear science as the unifying theme**, based on common concepts which unify a broad range of problems of basic science, such as pattern formation and control, high-dimensional dynamics, turbulence/stochasticity, and chemical sensing and communication between aquatic organisms living in a turbulent flow environment. Research and advanced training will be cross-disciplinary, emphasizing the diverse range of applications in many fields, from biology and neuroscience to engineering problems involving fluids, interface motion, flows at nanoscales, and nonlinear control.
- **Training in cross-cutting methodologies**, stimulating cross-disciplinary research and communication skills through a balanced core curriculum that includes both lecture-based and project-based courses. A key innovation is an early research experience, with small teams investigating topics guided by faculty members with complementary perspectives. The IGERT will form the core of a vibrant, highly cooperative environment through a combination of cross-departmental student-run seminars, regional workshops, and a visitor program. Global perspective will be enhanced through internships at, and schools organized with sister institutions abroad.
- **Outreach**: GT has a notable record in minority recruitment and retention [2]; IGERT would seek to encourage a career choice in science or engineering, by involving undergraduates from Atlanta-area colleges (Clark-Atlanta, Morris-Brown, Morehouse and Spellman) in the IGERT summer research projects. Educational outreach will be also served by the development of a web-based advanced nonlinear science course.
- **Strategic impact**: GT is the premier science and technology institution in the Southeast, with strong state and industrial support. Already possessing a leading program in nonlinear science, GT teams up faculty in cross-disciplinary facilities and has moved aggressively into several allied areas, establishing new centers in nonlinear science, bioinformatics, biotechnology, computing, and the Biomedical Engineering Department, unique in that it is run jointly with Emory University Medical School.
  IGERT SINC will offer advanced nonlinear science training and a highly cooperative and cross-disciplinary environment to IGERT Fellows A coherent program which synthesizes research, lecture, workshop, and internship components will replace rigid major/minor disciplinary boundaries of a traditional graduate degree program.

c. Major Research Efforts

Nonlinear science is a rich subject, already sufficiently developed to form the core of the training proposed under this IGERT. However, available tools largely fail for systems involving many strongly coupled degrees of freedom. Conceptually new methods applicable to high numbers of deterministic degrees of freedom as well as to mixed systems of chaotic, integrable, and stochastic components need to be developed; the present dearth of such methods remains a fundamental barrier for the direct application of the concepts of nonlinear science to many fields.

This IGERT will seek to break this impasse by bringing together teams of IGERT Fellows and faculty with complementary skills; experimental, numerical and theoretical. In what follows the core faculty listed in sect. (C).a are indicated by initials, (i.e. [LAB] for L.A. Bunimovich). The IGERT will actively seek participation of GT nonlinear sciences faculty across the campus in Fellow co-advising, as well as other key IGERT activities. Not divisible into neatly disentangled Major Research Efforts, all research under this IGERT addresses the same grand challenge of high-dimensional dynamics along two interwoven major themes: (1) the interplay between *stochasticity and nonlinearity*, and (2) the role of *patterns and coherent structures in high-dimensional dynamics*.

Stochasticity & nonlinearity, or "getting messages across in a turbulent world": As objects move through fluids, they shed complex but characteristic wakes [JY,FS]. For aquatic organisms, these coherent patterns may reveal the identity of a prey, predator, or mate; the identification taking place in turbulent sea, with characteristic hydrodynamic turbulence of size comparable to the biologically induced coherent structures. Rather than interfering with sensory perception, this turbulence sometimes seems to enhance it. How do such organisms recognize biologically-created flows, distinguish them from background turbulence? One surprising answer comes from what initially was a basic discovery [4] in nonlinear science [KW]. In vitro experiments [5] showed that hair cells exhibit *stochastic resonance*, a counterintuitive nonlinear phenomenon where detection of weak signals is *enhanced* by the presence of external noise. From experimental and theoretical points of view, hair cells are a particularly apt testbed, both for the precise measurements of influence of noise, and because the physiology of hair cells is so well characterized that it is possible to write down accurate equations describing their dynamics.

Figure 1: Planktonic copepods accelerate their swimming appendages up to $50m/s^2$. Two vortices are created by 4 pairs of legs galloping twice within the field of view [3].

Turbulence thus not only fails to obscure the message, but can help get it across, not only for living organisms but also in environmental science (pollutant dispersion, mixing pesticide dispersal [PKY,FS]). Similar challenges arise in the study of the brain activity during epileptic seizures: neuronal noise masks nonlinear dynamics of comparable magnitude [PC]. Put succinctly, a central question across many disciplines, and one of the driving questions motivating this IGERT is: *"How do sensory systems manage to be so sensitive in a world so noisy?"*

This challenge simply cannot be addressed from the confines of a traditional departmental PhD program, specialty-specific funding. Numerical analysis of turbulent flow, experimental detection of signal transmission through fluid media, neurophysiological and behavioral studies of living organisms, and information processing of signals acquired by sensory arrays are all required and can be provided only by cross-disciplinary teams.

Here the proposed IGERT will foster the necessary collaborations. It will draw inspiration from the already running GT "Signals in the Sea" (SiS) IGERT (NSF award #0114400) which focuses on empirical biology data and field work, synergistically adding theoretical training to the SiS IGERT Fellows. Conversely, the insights of SiS biologists will provide invaluable stimulus to this IGERT. The computationally demanding investigations of turbulent fluid flow [PKY,FS] will be essential to addressing the fluid dynamics aspects. For example, Lagrangian dynamics makes it possible to understand the fact that even stationary fluid flows can give rise to chaotic particle paths. In this context, ergodic theory has already inspired a non-intrusive experimental technique [6] for visualizing 3D fluid flows [FS]; in neurodynamics of epileptic focii, ergodic theory is the basis for new tools for analysis of very noisy neuronal data [7]. This IGERT offers a unique angle of attack on such problems, through hybrid systems which are part living tissue and part electronic circuitry.

"Riding the wave" or high-dimensional dynamics: Hybrid simulations/experiments of neurons [8], and thermally actuated control of thin films [9] pioneered here [RJB,MS] both rely on modern electronics being much faster than the neuronal or fluid dynamics. The RJB group, with expertise in real-time programming and cellular electrophysiology, can both observe and simulate in real time small neuronal systems. Were the theoretical models of comparable sophistication, this strategy could be reversed, and the real-time simulations could be used to *control* spatiotemporal patterns, neuronal ensembles, perhaps even entire organs. The IGERT teams need to quantify the dynamics underlying synchronization between ensembles of neurons and interaction of coherent structures in turbulent flows; the IGERT will address these challenges relying on hybrid neuronal and other experiments, such as the real-time 3D optical measurements of wakes in fluids [3], and fluid dynamic simulations of such wakes [6]. This wealth of observations drives the other theme of this IGERT: *"How to characterize and control high-dimensional, spatially extended, and very noisy dynamics?"*

Spatially extended systems are systems with very many coupled degrees of freedom whose dynamics range from ordered to very disordered and turbulent. Theoretical challenges in learning to control such systems require understanding the instability mechanisms arising from the interplay between nonlinearity, transient dynamics, and stochasticity, often resulting in rather surprising dynamics. For instance, the onset of turbulence observed in shear flows occurs well within the Reynolds number regime, where the laminar flow is still linearly stable. Transient amplification in such systems leads to extreme sensitivity to noise, precluding any spatially localized control, even at physically unrealistic low levels of noise [10].

A theorist exploring classical field theories (for instance hydrodynamics, or systems modeled by the Ginzburg-Landau equations) faces a truly bewildering wealth of solutions. In order to understand how such systems go turbulent, one needs to determine, classify, and assign relative importance to the coherent structures they exhibit [11]. This research is still in its infancy, but it has led to a working hypothesis that the dynamics explores an approximate *finite* alphabet of unstable recurrent patterns, a hypothesis which IGERT SINC is uniquely poised to explore. On one hand the periodic orbit theory [12] offers the mathematical implementation of this intuitive picture [PC,FB,LAB], and on the other, the MS group possesses a unique skill: by means of their multipoint thermal actuation technique, they are able to *design* spatio-temporal patterns, and thus create and test experimentally the patterns singled out by the theory.

Why is IGERT essential to the success of the proposed research training? IGERT SINC will provide the kind of PhD support not available from individual grants, but vitally important to initiating, stimulating and sustaining cross-disciplinary research.

Experience from sister programs already demonstrates that IGERTs are very attractive to the best graduate students, as IGERT Fellows are given a great deal of independence in choosing co-advisers, and pursuing collaborative research projects nationally and internationally. IGERT SINC will seed the infrastructure that should outlive the NSF grant, supporting the new research directions through workshops and visitors, both from other nonlinear science research centers, and from institutions which could potentially apply the IGERT fostered methodologies to outstanding challenges in other disciplines. At present, no framework provides such a meeting place for the advancement of our cross-disciplinary research goals.

IGERT SINC will address the grand challenge of nonlinear science: Explore experimentally and describe theoretically the dynamics of high-dimensional nonlinear, noisy systems. Furthermore, this IGERT will foster applications of the methods thus developed to problems in engineering and biosciences.

d. Education and Training

In order to address the nonlinear science challenges detailed above, graduate education must transcend traditional departmental boundaries and produce a new breed of scientists of necessary breadth and versatility.

The proposed IGERT training initiative will emphasize the integration of knowledge and foster research independence and self-reliance through a nonlinear science curriculum based on novel lecture-based / project-based courses as well as co-advised independent research. Combined with seminars, internships, workshops and international schools this curriculum will equip IGERT Fellows with the methodology, basic tools and techniques of nonlinear science, as well as an understanding of their applications to specific problems of fundamental and practical significance.

Competitive IGERT Fellowships will be offered to outstanding students pursuing a minor in Nonlinear Science in conjunction with the PhD degree program in Fellow's home department. The IGERT four-level training sequence, detailed below, is coordinated with IGERT Fellow's home departments core requirements:

Level 1: Introduction to Nonlinear Science
Incoming IGERT Fellows will be introduced to the program with a **Welcoming Workshop/Retreat**, before the start of each Fall semester. Incoming Fellows will pair up with a nonlinear science advisor, external to her/his home department, who will oversee the Fellow's progress in tandem with a departmental advisor. In the Spring semester of the first year, Fellows will begin coursework with a lecture-based offering that introduces the mathematical and computational techniques of nonlinear science. This course will provide IGERT Fellows with the knowledge of basic concepts of nonlinear science (dynamics, stability, bifurcations) and establish a common language bridging the linguistic barriers between disciplines.

Level 2: Applications of Nonlinear Science.
Targeted primarily at the second-year students, this rotating sequence of courses will be based on the research interests of individual faculty and cover applications of the tools and techniques of nonlinear science to problems in physics, chemistry, biology, materials science, and engineering. Examples might include pattern formation, ecological models, turbulence and mixing, or dynamics of neuronal networks. Offered every Fall semester, this advanced course can be taken by IGERT Fellows more than once as the topics will change from year to year. The main objectives of this lecture-based course is to demonstrate how the common toolkit of nonlinear science can be used to address different disciplinary problems and to develop initial communication skills necessary for interdisciplinary research. Level 1 and 2 precursor courses [13] are already attracting a wide spectrum of students.

Level 3: Advanced Topics in Nonlinear Science.
Offered in the Spring semester, this project-based course will stress the development of communication skills. The main objectives will be to provide the second-year students with the collaborative skills and develop the independent learning ability necessary for genuine cross-disciplinary research. The course will be more structured than an independent project would be; with the research focusing on specific projects carried out by small IGERT Fellow teams, supervised by faculty members with complementary expertise and perspectives. The faculty will select the problems and advise the Fellows on the approaches an experienced researcher would employ in similar research settings. A typical project will have a well documented research literature core, leading up to open research questions. Proposed topics include: synchronization in arrays of live and artificial neurons; pattern formation, selection, and control; mixing and stirring by coherent structures in turbulent flows; spatiotemporal dynamics of communicating ecological communities.

Level 4: Independent Research
By the third year IGERT Fellows will be expected to have started independent research. This phase of the training program will emphasize the integration of research expertise from several disciplines, for solution of problems lying at their interface. In order to ensure and nurture the cross-disciplinary focus of their research program, all IGERT Fellows will be co-advised, with one advisor from the Fellow's home department and the other one external.

A **Graduate Student Seminar** organized for and by the advanced students will give the IGERT Fellows an opportunity to present their research in a supportive setting, and enhance their communication skills. While the ultimate responsibility for teaching a student the ethical way to conduct research lies with the advisor, this graduate seminar, sometimes run jointly with the parallel GT SiS IGERT ethics seminar, and sometimes advised by the GT Ethics Officer, will also provide a forum for directed discussions on ethics and conflicts of interest in research.

IGERT internships will offer the Fellows participation in collaborative cross-disciplinary research at the highest level, international and national. Perhaps more so than any other discipline, nonlinear science is driven by the flux of ideas across disciplinary boundaries. Given the essential role students at all levels play in conducting research, such flux is most effectively nurtured through a cross-disciplinary training program which integrates research, education, and training. In cases where the thesis research clearly warrants it, IGERT will sponsor 3-6 month Fellow internships at international or national nonlinear science sister centers [15] [16], and will in return host interns sent to GT by sister centers. Internships will, on the one hand, offer the Fellows invaluable expertise needed in their research and, on the other hand, bring into IGERT foreign interns, thus enhancing (nonlinearly!) the quality of IGERT Fellows research environment.

While the nationally highly ranked IGERT faculty has broad expertise across several branches of nonlinear science, many research projects will require expertise and/or collaborations not available locally within GT. Much of the cross-disciplinary GT research already has a significant international component, and an intern might be hosted by the Bayreuth University, the Niels Bohr Institute (Copenhagen), the Max-Planck-Institute for Complex Systems (Dresden), or another sister center, in order to receive additional training in experimental or theoretical pattern formation, granular media and complex fluids programs, or perhaps study an organism not available in the US.

The courses and projects will envelop IGERT Fellows in an intellectual community that stresses commonalities among disciplines. As the level of courses described above will vary from advanced undergraduate to second-year graduate, the sequence will target the range of students, both inside and outside of the IGERT program. The recruiting efforts will also benefit from another IGERT initiative, a web-based year-long advanced graduate nonlinear course [12], reaching students well beyond GT. The IGERT competitive Fellowship will assist in recruitment of the very best students, making careers in science/engineering more attractive to a wider range of students. IGERT Fellowships will be used exclusively to support co-advised Fellows, enabling them to concentrate on the study and research already during the first two years of their PhD programs.

In order to offer students a deep learning experience outside the PhD thesis research, the educational core of the training program will be supplemented by the further key components:

An **Interdisciplinary Nonlinear Science Seminar series** [14], initiated in January 2001, draws a wide attendance across GT and other Atlanta based institutions. IGERT would broaden the scope to engineering and biological applications.

A **visitor program** to promote and maintain national, international, and industrial collaborations, with priority given to visitors whose research demonstrate potential as external mentors to IGERT fellows.

An annual **Regional Nonlinear Workshop** organized jointly with other Southeastern universities (Duke, U. of Alabama, U. of Florida) will expose students to the cutting edge research, and give them an opportunity to present their own work.

A **Nonlinear Science School**, a bi-yearly summer school program hosted by the IGERT in conjunction with the CNS and a sister international center, will start with a sequence of tutorial-style lectures, preparing IGERT Fellows as well as students from other participating programs for the research level part of the School. For instance, IGERT proposes to host a summer school in the "Stochastic nonlinear processes in biology" series, sponsored by NIH, either in Atlanta or at the Centro Internacional de Ciencias, Cuernavaca, Mexico [15].

e. Management, Assessment, and Institutional Commitment

The director of the proposed IGERT: School in Nonlinearity & Complexity program is P. Cvitanović (School of Physics). Prior to moving to GT, Cvitanović co-founded and directed (1993-1998) the Center for Chaos and Turbulence Studies [17] at the Niels Bohr Institute, Copenhagen, a leading international center *par excellence*. PC led the initiative to create a Center for Complex Systems at the Northwestern University, and was the PI on the NSF "IGERT: Complex Systems in Science and Engineering" program, awarded to Northwestern in 2000 (see sect. C.g). He currently holds the Glen Robinson Chair in Nonlinear Sciences and is the director of the newly created Center for Nonlinear Science.

Due to the full cross-disciplinary integration of the proposed IGERT, the day-to-day training, research, and other activities of the program will be supervised by a single, annually faculty elected 5-person *Executive Committee*, which will administer fellowships, internships, visitor invitations, and other IGERT resources in accordance with the goals of the program. Faculty status implies no entitlement to IGERT resources; the Executive Committee will base its decisions solely by proposal's quality and cross-cutting impact.

The evaluation of the program will rely on detailed feedback from students, internship hosts, faculty, visitors, and thesis advisors. The program will assess annually the performance of the IGERT Fellows, interns, research progress, nonlinear science course sequence, seminar, visitor program, summer school and Regional Conference. Starting with the third year, the assessment will also involve the IGERT sponsored research impact, recruitment, retention, PhD theses time-to-degree measures, publications, and a visit of an external expert committee.

f. Expected Resource Commitments

GT faculty in nonlinear sciences was ranked 5th nationally in the most recent U.S. News and World Report survey (1999). Its commitment to this field is demonstrated by recent chaired faculty appointment in physics (PC), continued recruiting efforts in mathematics and physics (junior and/or senior experimental nonlinear physics searches in 2001-03), and the expansion in biosciences, including a biophysics chair search.

GT encourages cross-departmental and cross-disciplinary research training. The *GT Center for Nonlinear Science* (CNS) which began operation July 2001, already spans nonlinear science efforts across eight science and engineering departments [18]. The IGERT office and the common meeting room will be housed in CNS, Howey Physics building. Shared office space and computer facilities will be provided to the IGERT Fellows, with the IGERT experimental training laboratories housed in proximity to the participating faculty laboratories. An experimental computation laboratory is operational in the Math building [19]. The IGERT will require substantial parallel processor computing resources to implement its objectives. GT's CCMST, (its co-director RH is on the IGERT faculty) already provides a 72-processor IBM SP2. Up to one third of this resource will be available to the IGERT Fellows.

The Deans of the participating Colleges and the Institute Vice-President for Research and Dean of Graduate Studies support the initiative and are already funding the CNS for an initial three-year period. As a seed cost-sharing contribution, GT has committed $147K/year for the fiscal years 2002-2004, providing funding for 2-3 postdoctoral research associates, a distinguished lecturer series, visitors, workshops, the Nonlinear Science seminars, secretarial and part-time computer/web support, and ca. $80K in the annual Nonlinear Sciences Chair endowment, and $30K for equipment funds to cover the equipping of research associates' offices, research infrastructure including a Linux network, a platform for intensive computation, and office equipment.

In addition, if this proposal is funded, GT will support additional graduate students in the IGERT program (the precise number to be negotiated), cover the differential tuition for out-of-state graduate students, and match the capital equipment expenditure of the IGERT experimental and computational training program.

g. Recent Traineeship Experience and Results from Prior NSF Support

Two prior NSF grants form the basis for this IGERT preproposal: P. Cvitanović, PI (until moving to GT) - **IGERT #9987577**: Northwestern University *Dynamics of Complex Systems in Science and Engineering* (2000-2004), and L.A. Bunimovich, CoPI and Director - **GIG #9632032**: *Southeast Applied Analysis Center* (1996-2001).

SAAC [20] has been very successful in establishing regional educational initiatives (detailed in sect. (C).d) which the proposed IGERT intends to extend and build upon.

PC follows closely activities of the Northwestern IGERT, and the proposed IGERT would interact closely with the NWU sister program, with the GT SiS IGERT, and the Arizona and Cornell IGERT programs. Significant aspects of this proposal already are built upon our colleagues' IGERT experiences. The student satisfaction with the programs is high, and they are an invaluable resource in breaking down departmental walls. PC has moved to GT because of its commitment to nonlinear science (see sect. C.f), because of its requirement for minor in PhD studies actively fosters cross-disciplinarity, and because the departments facilitate cross-disciplinary teaching.

European PhD programs typically require that their students spend 3-12 months abroad, with support from their home institutions. As the European Dynamics Days Advisory Committee Secretary, PC has close working relations with the leading nonlinear science centers in Europe and elsewhere. Foreign interns hosted so far have been of significant benefit both to PC's research program, as well as to US students who interacted with the interns. IGERT will provide a framework for enhancing these student exchanges.

3 References

(D) References Cited

[1] "Physics in a new era: an overview," *National Research Council report* (National Academy Press, Washington, D.C., 2001); www.nap.edu/books/0309073421/html/.

[2] Interview with GT President Clough, *Technique* (March 22, 2002); www.cns.gatech.edu/CNS/Clough.html.

[3] J. Yen, "Life in transition: balancing inertial and viscous forces by planktonic copepods," *Biol. Bull.* **198**, 213 (2000).

[4] K. Wiesenfeld and F. Moss, "Stochastic resonance and the benefits of noise: from ice ages to crayfish and SQUIDs," *Nature* **373**, 33 (1995).

[5] F. Jaramillo and K. Wiesenfeld, "Mechanoelectrical transduction assisted by Brownian motion: a role for noise in the auditory system," *Nature Neuroscience* **1**, 384 (1998).

[6] I. Mezić and F. Sotiropoulos, "Ergodic Theory and Experimental Visualization of Invariant Sets in Chaotically Advected Flows," *Physics of Fluids* **14**, 2235 (2002).

[7] M.W. Slutzky, P. Cvitanović and D.J. Mogul, "Deterministic chaos and noise in three in vitro hippocampal models of epilepsy," *Annals of Biomedical Engineering* **29**, 1 (2001).

[8] C.G. Wilson, R.J. Butera, C.A. DelNegro, J. Rinzel, and J.C. Smith. "Interfacing computer models with real neurons: respiratory 'cyberneurons' created with the dynamic clamp," *Advances in Experimental Biology and Medicine* **499**, 119 (2001).

[9] D. Semwogerere and M.F. Schatz, "Evolution of hexagonal patterns from controlled initial conditions in a Bénard-Marangoni convection experiment," *Phys. Rev. Lett.* **88**, (2002).

[10] R.O. Grigoriev and A. Handel, "Spectral Theory for the Failure of Linear Control in a Nonlinear Stochastic System," submitted to *Phys.Rev.E.*

[11] F. Christiansen, P. Cvitanović, and V. Putkaradze, "Spatiotemporal chaos in terms of unstable recurrent patterns," *Nonlinearity* **10**, 1 (1997).

[12] P. Cvitanović *et al.*, *Classical and Quantum Chaos*, advanced graduate course, available electronically at www.cns.gatech.edu/ChaosBook/.

[13] www.cns.gatech.edu/courses

[14] www.cns.gatech.edu/research.

[15] www.cns.gatech.edu/centers/sorelle.html.

[16] www.cns.gatech.edu/centers/sisters.html.

[17] Center for Chaos And Turbulence Studies, www.nbi.dk/CATS.

[18] Center for Nonlinear Science, www.cns.gatech.edu.

[19] Analysis, Computation, and Experiment Lab, www.ace.gatech.edu.

[20] The Southeast Applied Analysis Center, www.math.gatech.edu/saac.