(C) Project Description:

a. List of Participants

Director:

Predrag Cvitanović, Glen Robinson Chair, Professor, School of Physics (COS)

Executive Committee:

Leonid A. Bunimovich, Regents' Professor, School of Mathematics (COS)
Kurt Wiesenfeld, Professor, School of Physics (COS)
Jeannette Yen, Professor, School of Biology (COS)
P.K. Yeung, Associate Professor, School of Aerospace Engineering (COE)

Faculty:

Y.H. Chen, Associate Professor, School of Mechanical Engineering (COE)
R.F. Fox, Regents' Professor, Chair of School of Physics (COS)
R. Grigoriev, Assistant Professor, School of Physics (COS)
E.M. Harrell, Professor, School of Mathematics(COS)
C.A. Klausmeier, Assistant Professor, School of Biology (COS)
P.J. Mucha, Assistant Professor, School of Mathematics (COS)
G.P. Neitzel, Professor, Mechanical Engineering (COE)
E. Riedo, Assistant Professor, School of Physics (COS)
M. Schatz, Associate Professor, School of Physics (COS)
L.A. Smith, Professor, School of Earth and Atmospheric Sciences (COS)
F. Sotiropoulos, Associate Professor, School of Civil & Environmental Engineering (COE)
H.M. Zhou, Assistant Professor, School of Mathematics (COS)

The key to the school and departmental abbreviations used throughout the proposal:

GT	Georgia Institute of Technology
COS	College of Sciences
COE	College of Engineering
Bio	School of Biology
AE	School of Aerospace Engineering
CE	School of Civil & Environmental Engineering
EAS	School of Earth and Atmospheric Sciences
Math	School of Mathematics
ME	School of Mechanical Engineering
Phys	School of Physics

b. Vision, Goals and Thematic Basis

Nonlinear science impacts a broad spectrum of disciplines: for example, the Topical Group on Statistical and Nonlinear Physics and the Activity Group on Dynamical Systems are today among the most active within APS and SIAM, respectively. However, today nonlinear science also faces very hard challenges. In a 2001 report "Physics in a New Era: an Overview" [1] the National Research Council has identified six "grand challenges". Three of the six – "understanding complex systems", "applying physics to biology", and "creating new materials" – demand conceptually new methods for dealing with systems with many degrees of freedom in strongly stochastic environments. These are problems with no clear boundaries between physics, mathematics, engineering and life sciences.

Only a broad cross-disciplinary effort can make headway here. In order to prepare young researchers for these challenges, we propose to enrich their education by offering training in the tools and methods of nonlinear science that bridges departmental boundaries, academic and industrial research. We envision the IGERT **Dynamics in Many Dimensions** program, fostering

- **Cross-cutting research initiatives, with nonlinear science as the unifying theme**, based on common concepts which unify a broad range of problems in basic science. GaTech has an exceptionally strong community of experts in high-dimensional dynamics: fluid mechanics, pattern formation and control, turbulence, stochasticity, interface motion, nanofluidics, nonlinear control, and dynamics of organisms living in turbulent flow environments. Research and advanced training will be cross-disciplinary, emphasizing the diverse range of applications in many fields, from math and physics to engineering.
- **Training in cross-cutting methodologies**, stimulating cross-disciplinary research and communication skills through a balanced core curriculum that includes both lecture-based and project-based courses. A key innovation is an early research experience, invovling small teams investigating topics guided by faculty members with complementary perspectives. The IGERT program will form the core of a vibrant, highly cooperative environment through a combination of cross-departmental student-run seminars, regional workshops, and a visitor program. Global perspective will be enhanced through internships and summer schools with sister institutions abroad.
- **Outreach**: GaTech has a notable record in minority recruitment and retention [2]; the IGERT program will encourage undergraduates from historically black Atlanta-area colleges (Clark-Atlanta, Morris-Brown, Morehouse and Spelman) to chose a career in science or engineering, by involving them in IGERT summer research projects. Educational outreach will be also served by the development of a web-based advanced nonlinear science course.
- **Strategic impact**: GaTech is the premier science and technology institution in the Southeast, with strong state and industrial support. Already possessing a leading program in nonlinear science, GaTech teams up faculty in cross-disciplinary facilities and is moving aggressively into several allied areas, establishing new centers in nonlinear science, bioinformatics, biomedical engineering, and large scale computing.

The IGERT program will offer advanced nonlinear science training and a highly cooperative and cross-disciplinary environment to IGERT Fellows. A new program which synthesizes research, lecture, workshop, and internship components will enrich their more narrow graduate degree programs.

c. Major Research Efforts

Methodology of nonlinear science forms the core of the training proposed under this IGERT program. However, available tools largely fail for systems involving many strongly coupled degrees of freedom. Conceptually new methods applicable to high numbers of deterministic degrees of freedom as well as to mixed systems of chaotic, integrable, and stochastic components need to be developed; the present dearth of such methods remains a fundamental barrier for the direct application of nonlinear science to many important outstanding problems.

The IGERT will seek to break this impasse by bringing together teams of IGERT Fellows and faculty with complementary skills; experimental, numerical and theoretical. In what follows, the core faculty listed in sect. (C).a are indicated by initials, (i.e. [LAB] for L.A. Bunimovich). The GaTech nonlinear sciences faculty across the campus will take part in Fellow co-advising, as well as other key IGERT activities. Not divisible into neatly disentangled Major Research Efforts, all research under this IGERT addresses the same grand challenge of high-dimensional dynamics: the role of *patterns and coherent structures in high-dimensional dynamics*, and the interplay between *stochasticity and nonlinearity*.

High-dimensional dynamics – observations: In their research, the IGERT faculty explores a broad range of high-dimensional phenomena, on scales ranging from very large to very small. For example:

In medium term climate forecasts, LAS is developing methods which aim to achieve optimal weather prediction in the face of real, noisy data, with dynamical models that account for both chaotic and stochastic aspects of climate dynamics. PKY carries out some of nation's largest supercomputer numerical simulations of the exact Navier-Stokes equations, employing more than 10^9 grid points, in order to elucidate turbulent diffusion and mixing in the atmosphere [3]. CAK seeks to understand the principles underlying the striking patterns in space, time, and organization observed in ecosystems. PJM's simulations of sedimenting suspensions reveal surprisingly rich dynamics. FS's Navier-Stokes simulations of a hydro-mechanical "fish" reveal beautiful unstable vortices [4]. JY's copepods generate characteristic vortex wakes [5], see fig. 2. On still smaller scale, in their development of "opto-microfluidics", MS and RG observe good mixing in optothermally manipulated microdroplets, even though no such mixing is expected [6]. At atomic scales ER observes counterintuitive nanofluidic behaviors with implications for design of MEMS, "labs on chip" and cell manipulation. What all of these puzzles have in common is that they are manifestations of dynamics of spatially extended systems, systems with very many coupled degrees of freedom. This wealth of observations provides the unifying focus of this IGERT: *"How to characterize and control high-dimensional, spatially extended, and very noisy dynamics?"*

High-dimensional dynamics – challenges: Numerical explorations of systems such as hydrodynamics reveal a truly bewildering wealth of solutions. Turbulent systems never settle down, but we can identify a snapshot as a "cloud'. How do we do it? In order to understand how such systems go turbulent, one needs to determine, classify, and assign relative importance to the coherent structures they exhibit [7]. This research is still in its infancy, but it has led to a working hypothesis that the dynamics explores an approximate *finite* alphabet of unstable recurrent patterns, a hypothesis which the IGERT program is uniquely poised to explore. On the one hand, periodic orbit theory [8] offers the mathematical implementation of this intuitive picture [PC,LAB]. On the other hand, the MS group possesses a unique skill: by means of their thermally actuated control of thin films [9] pioneered here [MS] they are able to *design* spatio-temporal patterns, and thus create and test experimentally the patterns singled out by the theory. The IGERT teams need to quantify the dynamics underlying interaction of coherent structures in turbulent flows; the IGERT will address these challenges relying on experiments such as the real-time 3D optical measurements of wakes in fluids [JY], and fluid dynamic simulations of such wakes [FS].

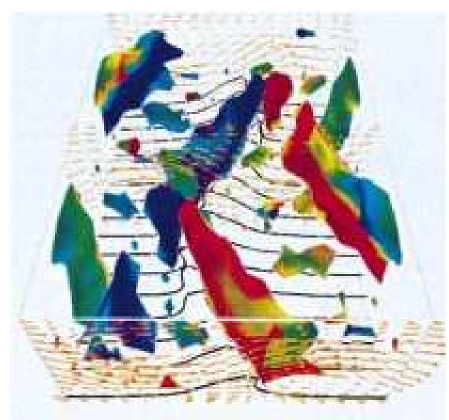

Figure 1: *A 3-d instantaneous snapshot of an unstable periodic solution embedded in Couette turbulence [10]. Vectors and color code velocities and vorticities.*

The experience and skills of IGERT's **computational fluid dynamicists** are absolutely essential for any further progress. Pushing the current limits of what is numerically attainable in solving large sets of PDEs, the best we currently can do is write down a set of hydrodynamical PDEs, discretize it on a mesh or a basis function set on the order of 10^4-10^9 components, and follow trajectories in high-dimensional phase spaces. An example of what has been attained so far is the calculation by Kawahara and Kida [10]: the first demonstration of existence of an unstable recurrent structure in a turbulent hydrodynamic flow, in a *15,422-dimensional* numerical discretization of three-dimensional plane Couette turbulence at $Re = 400$, see fig. 1.

Theoretical challenges in analysis and learning to control such systems require understanding the instability mechanisms arising from the interplay between nonlinearity, transient dynamics, and stochasticity, often resulting in rather surprising dynamics. For instance, the onset of turbulence observed in shear flows occurs well within the Reynolds number regime, where the laminar flow is still linearly stable. Transient amplification in such systems leads to extreme sensitivity to noise, precluding any spatially localized control, even at physically unrealistic low levels of noise [11].

Stochasticity & nonlinearity: Even in a turbulent sea, a plankton can identify a prey, predator, or mate by the complex but characteristic wakes that bodies moving through fluid shed [JY,FS]. How do such organisms recognize biologically-created flows, distinguish them from background turbulence of comparable characteristic size? One surprising answer comes from what initially was a basic discovery [12] in nonlinear science [KW]. In vitro experiments [13] showed that hair cells exhibit *stochastic resonance*, a counterintuitive nonlinear phenomenon where detection of weak signals is *enhanced* by the presence of external noise.

Figure 2: Planktonic copepods accelerate their swimming appendages up to $50 m/s^2$. Two vortices are created by 4 pairs of legs galloping twice within the field of view [5].

Thus turbulence not only fails to obscure the message, but can help get it across, not only for living organisms but also in dispersal of pollutants through atmosphere [PKY,FS]. A central question across many disciplines, and one of the driving questions motivating this IGERT is: *"How do unstable coherent structures manage to be so recognizable in a world so noisy?"*

Why is IGERT essential to the success of the proposed research training? The challenge of high-dimensional dynamics simply cannot be addressed from the confines of a traditional departmental PhD program. While the IGERT faculty representing such diverse research efforts already offers a vibrant intellectual environment that spans many sciences and engineering departments, the IGERT program is greatly needed in order to provide the kind of PhD support not available from individual grants, but vitally important to initiating, stimulating and sustaining cross-disciplinary research. Numerical analysis of turbulent flow [PKY,FS], experimental detection of signal transmission through fluid media [JY], and information processing of signals acquired by sensory arrays are all required and can be provided only by cross-disciplinary teams. Here the proposed IGERT will foster the necessary collaborations.

Experience from sister programs already demonstrates that IGERTs are very attractive to the best graduate students, as IGERT Fellows are given a great deal of independence in choosing co-advisers, and encouraged to take part in collaborative research projects nationally and internationally. This IGERT program will seed the infrastructure that should outlive the NSF grant, supporting the new research directions through workshops and visitors, both from other nonlinear science research centers, and from institutions which could potentially apply the IGERT fostered methodologies to outstanding challenges in other disciplines. At present, no framework provides such a meeting place for the advancement of our cross-disciplinary research goals.

This IGERT program will address the grand challenge of nonlinear science: Explore experimentally and describe theoretically the dynamics of high-dimensional nonlinear, noisy systems. Furthermore, this IGERT will foster applications of the methods thus developed to problems in engineering and biosciences.

d. Education and Training

In order to address the nonlinear science challenges detailed above, graduate education must transcend traditional departmental boundaries and produce a new breed of scientists of necessary breadth and versatility.

The proposed IGERT training initiative will emphasize the integration of knowledge and foster research independence and self-reliance through a nonlinear science curriculum based on novel lecture-based / project-based courses as well as co-advised independent research. Combined with seminars, internships, workshops and international schools this curriculum will equip IGERT Fellows with the methodology, basic tools and techniques of nonlinear science, as well as an understanding of their applications to specific problems of fundamental and practical significance.

Competitive IGERT Fellowships will be offered to outstanding students pursuing a minor in Nonlinear Science in conjunction with the PhD degree program in each Fellow's home department. The IGERT four-level training sequence, detailed below, is coordinated with the IGERT Fellow's home department core requirements:

Level 1: Introduction to Nonlinear Science

Incoming IGERT Fellows will be introduced to the program with a **Welcoming Workshop/Retreat**, before the start of each Fall semester. Incoming Fellows will pair up with a nonlinear science advisor, external to her/his home department, who will oversee the Fellow's progress in tandem with a departmental advisor. In the Spring semester of the first year, Fellows will begin course work with a lecture-based offering that introduces the mathematical and computational techniques of nonlinear science. This course will provide IGERT Fellows with the knowledge of basic concepts of nonlinear science (dynamics, stability, bifurcations) and establish a common language bridging the linguistic barriers between disciplines.

Level 2: Applications of Nonlinear Science.

Targeted primarily at the second-year students, this rotating sequence of courses will be based on the research interests of individual faculty and cover applications of the tools and techniques of nonlinear science to problems in physics, chemistry, biology, materials science, and engineering. Examples might include pattern formation, ecological models, or turbulence and mixing. Offered every Fall semester, this advanced course can be taken by IGERT Fellows more than once as the topics will change from year to year. The main objectives of this lecture-based course is to demonstrate how the common toolkit of nonlinear science can be used to address different disciplinary problems and to develop initial communication skills necessary for interdisciplinary research. Level 1 and 2 precursor courses [14] are already attracting a wide spectrum of students.

Level 3: Advanced Topics in Nonlinear Science.

Offered in the Spring semester, this project-based course will stress the development of communication skills. The main objectives will be to provide the second-year students with the collaborative skills and develop the independent learning ability necessary for genuine cross-disciplinary research. The course will be more structured than an independent project would be; with the research focusing on specific projects carried out by small IGERT Fellow teams, supervised by faculty members with complementary expertise and perspectives. The faculty will select the problems and advise the Fellows on the approaches an experienced researcher would employ in similar research settings. A typical project will have a well documented research literature core, leading up to open research questions. Proposed topics include: synchronization in arrays of stochastic elements; pattern formation, selection, and control; mixing and stirring by coherent structures in turbulent flows; spatiotemporal dynamics of communicating ecological communities.

Level 4: Independent Research

By the third year IGERT Fellows will be expected to have started independent research. This phase of the training program will emphasize the integration of research expertise from several disciplines, for solution of problems lying at their interface. In order to ensure and nurture the cross-disciplinary focus of their research program, all IGERT Fellows will be co-advised, with one advisor from the Fellow's home department and the other one external.

A **Graduate Student Seminar** organized for and by the advanced students will give the IGERT Fellows an opportunity to present their research in a supportive setting, and enhance their communication skills. While the ultimate responsibility for teaching a student the ethical way to conduct research lies with the advisor, this graduate seminar, sometimes run jointly with the parallel GaTech IGERT ethics seminar, and sometimes advised by the GaTech Ethics Officer, will also provide a forum for directed discussions on ethics and conflicts of interest in research.

IGERT internships will offer the Fellows participation in collaborative cross-disciplinary research at the highest level, international and national. In conducting research, the flux of ideas across disciplinary boundaries is most effectively nurtured through a cross-disciplinary training program which integrates research, education, and training. In cases where the thesis research clearly warrants it, IGERT will sponsor

3-6 month Fellow internships at international or national nonlinear science sister centers [16, 17], and will in return host interns sent to GaTech by sister centers. Internships will, on the one hand, offer the Fellows invaluable expertise needed in their research and, on the other hand, bring into IGERT foreign interns, thus enhancing the quality of the IGERT Fellows' research environment.

While the nationally highly ranked IGERT faculty has broad expertise across several branches of nonlinear science, some of the research projects will require expertise and/or collaborations not available locally within GaTech. Much of the cross-disciplinary GaTech research already has a significant international component, and an intern might be hosted by Bayreuth University, the Niels Bohr Institute (Copenhagen), the Max-Planck-Institute for Complex Systems (Dresden), or another sister center, in order to receive additional training in experimental or theoretical pattern formation, and complex fluids programs.

The courses and projects will envelop IGERT Fellows in an intellectual community that stresses commonalities among disciplines. As the level of courses described above will vary from advanced undergraduate to second-year graduate, the sequence will target the range of students, both inside and outside of the IGERT program. The recruiting efforts will also benefit from another IGERT initiative, a web-based year-long advanced graduate nonlinear course [8], reaching students well beyond GaTech. The IGERT competitive Fellowship will assist in recruitment of the very best students, making careers in science/engineering more attractive to a wider range of students. IGERT Fellowships will be used exclusively to support co-advised Fellows, enabling them to concentrate on study and research during the first two years of their PhD programs. In order to offer students a deep learning experience outside the PhD thesis research, the educational core of the training program will be supplemented by the further key components:

An **Interdisciplinary Nonlinear Science Seminar series** [15], initiated in January 2001, draws a wide attendance across GaTech and other Atlanta based institutions. IGERT would broaden the scope to engineering and biological applications.

A **visitor program** to promote and maintain national, international, and industrial collaborations, with priority given to visitors whose research demonstrate potential as external mentors to IGERT fellows.

An annual **Regional Nonlinear Workshop** organized jointly with other Southeastern universities (Duke, U. of Alabama, U. of Florida) will expose students to the cutting edge research, and give them an opportunity to present their own work.

A **Nonlinear Science School**, a bi-yearly summer school program hosted by the IGERT in conjunction with the CNS and a sister international center, will start with a sequence of tutorial-style lectures, preparing IGERT Fellows as well as students from other participating programs for the research level part of the School. For instance, IGERT proposes to host a summer school in the "Stochastic and high-dimensional nonlinear processes", either in Atlanta or at the Centro Internacional de Ciencias, Cuernavaca, Mexico [16].

e. Management, Assessment, and Institutional Commitment

The director of the proposed Dynamics in Many Dimensions program is P. Cvitanović (School of Physics). In the period 1993-1998 Cvitanović founded and directed *Center for Chaos and Turbulence Studies* (CATS) at the Niels Bohr Institute, Copenhagen, a cross-disciplinary effort which became one of Europe's leading centers for nonlinear science, housing and in part funding approximately 15 faculty, 8 post-docs, 45 graduate students, 15 long term visitors, 40 short term visitors, and 5 workshops/conferences in any given year. Prior to moving to GaTech PC led the initiative to create a complex systems prgram at the Northwestern University, and was the PI on the NSF "IGERT: Complex Systems in Science and Engineering" program, awarded to Northwestern in 2000 (see sect. C.g). He currently holds the Glen Robinson Chair in Nonlinear Sciences and is the director of GaTech Center for Nonlinear Science.

Due to the full cross-disciplinary integration of the proposed IGERT, the day-to-day training, research, and other activities of the program will be supervised by a single, annually faculty elected 5-person *Executive Committee*, which will administer fellowships, internships, visitor invitations, and other IGERT resources in accordance with the goals of the program. Faculty status implies no entitlement to IGERT resources; the Executive Committee will base its decisions solely by proposal's quality and cross-cutting impact.

The evaluation of the program will rely on detailed feedback from students, internship hosts, faculty, visitors, and thesis advisors. The program will assess annually the performance of the IGERT Fellows, interns, research progress, nonlinear science course sequence, seminar, visitor program, summer school and Regional Conference. Starting with the third year, the assessment will also involve the IGERT sponsored research impact, recruitment, retention, PhD theses time-to-degree measures, publications, and a visit of an external expert committee.

f. Other Resources and Connections

GaTech faculty in nonlinear sciences was ranked 5th nationally in the most recent U.S. News and World Report survey (1999). GaTech's commitment to this field is reflected in the chaired faculty appointment in physics (PC), and continued recruiting efforts in mathematics and physics (junior and/or senior experimental nonlinear physics searches in 2001-04). As a seed contribution, GaTech committed $147K/year for the fiscal years 2002-2004, providing partial funding for 3-5 postdoctoral research associates working at CNS at any given time, a distinguished lecturer series, visitors, workshops, the Nonlinear Science seminars, center administrator, part-time computer/web support. Also ca. $80K is endowed annually for a Nonlinear Sciences Chair, and $30K was provided to cover the equipping of research associates' offices, research infrastructure (such as a Linux network which is a platform for intensive computation) and office equipment.

GaTech encourages cross-departmental and cross-disciplinary research training. The *GaTech Center for Nonlinear Science* (CNS) which began operation in 2001, already spans nonlinear science efforts across eight science and engineering departments [19]. The IGERT office and the common meeting room will be housed in CNS, Howey Physics building. Shared office space and computer facilities will be provided to the IGERT Fellows, with the IGERT experimental training laboratories housed in proximity to the participating faculty laboratories. An experimental computation laboratory is operational in the Math building. The IGERT program will require substantial parallel processor computing resources to implement its objectives. GaTech's Center for Computational Molecular Science and Technology, already provides a 72-processor IBM SP2. Up to one third of this resource will be available to the IGERT Fellows.

g. Recent Traineeship Experience and Results from Prior NSF Support

Two prior NSF grants form the basis for this IGERT preproposal: P. Cvitanović, PI (until moving to GaTech) - **IGERT #9987577**: Northwestern University *Dynamics of Complex Systems in Science and Engineering* (2000-2004), and L.A. Bunimovich, CoPI and Director - **GIG #9632032**: *Southeast Applied Analysis Center* (1996-2001).

Our faculty follows closely activities of the Northwestern, Arizona, Cornell and GaTech IGERT programs; significant aspects of this proposal incorporate our colleagues' IGERT experiences. The student satisfaction with the programs is high, and they are an invaluable resource in breaking down departmental walls.

European PhD programs typically require that their students spend 3-12 months abroad, with support from their home institutions. As the former Secretary of European Dynamics Days Advisory Committee, PC has close working relations with the leading nonlinear science centers in Europe and elsewhere. Foreign interns hosted so far have been of significant benefit both to PC's research program, as well as to US students who interacted with the interns. IGERT will provide a framework for enhancing these student exchanges.

(D) References Cited

[1] "Physics in a new era: an overview," *National Research Council report* (National Academy Press, Washington, D.C., 2001); www.nap.edu/books/0309073421/html/.

[2] Interview with GT President Clough, *Technique* (March 22, 2002); www.cns.gatech.edu/CNS/Clough.html.

[3] B.L. Sawford, P.K. Yeung, M.S. Borgas, P. Vedula, A. LaPorta, A.M. Crawford, and E. Bodenschatz, "Conditional and unconditional acceleration statistics in turbulence," *Phys. Fluids* **15**, 3478 (2003).

[4] F. Sotiropoulos, A. Gilmanov and E. Balaras, "A General Reconstruction Algorithm for Simulating Flows with Complex 3D Immersed Boundaries on Cartesian Grids," *Journal of Computational Physics* **191**, 660 (2003).

[5] J. Yen, "Life in transition: balancing inertial and viscous forces by planktonic copepods," *Biol. Bull.* **198**, 213 (2000).

[6] N. Garnier, R.O. Grigoriev and M.F. Schatz, "Optical manipulation of microscale fluid flow," *Phys. Rev. Lett.* **91**, 054501 (2003).

[7] Y. Lan and P. Cvitanović, "Variational method for finding periodic orbits in a general flow" *Phys. Rev.* **E 69**, 016217 (2004).

[8] P. Cvitanović *et al.*, *Chaos: Classical and Quantum*, advanced graduate course, available electronically at ChaosBook.org.

[9] D. Semwogerere and M.F. Schatz, "Evolution of hexagonal patterns from controlled initial conditions in a Bénard-Marangoni convection experiment," *Phys. Rev. Lett.* **88**, (2002).

[10] G. Kawahara and S. Kida, "Periodic motion embedded in plane Couette turbulence: regeneration cycle and burst," *J. Fluid Mech.* **449**, 291 (2001).

[11] R.O. Grigoriev and A. Handel, "Spectral Theory for the Failure of Linear Control in a Nonlinear Stochastic System," *Phys. Rev.* **E 66**, 065301(R) (2002).

[12] K. Wiesenfeld and F. Moss, "Stochastic resonance and the benefits of noise: from ice ages to crayfish and SQUIDs," *Nature* **373**, 33 (1995).

[13] F. Jaramillo and K. Wiesenfeld, "Mechanoelectrical transduction assisted by Brownian motion: a role for noise in the auditory system," *Nature Neuroscience* **1**, 384 (1998).

[14] www.cns.gatech.edu/courses

[15] www.cns.gatech.edu/research.

[16] www.cns.gatech.edu/centers/sorelle.html.

[17] www.cns.gatech.edu/centers/sisters.html.

[18] Center for Chaos And Turbulence Studies, www.nbi.dk/CATS.

[19] Center for Nonlinear Science, www.cns.gatech.edu.