Contact Line Instability and Pattern Selection in Thermally Driven Liquid Films

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Liquids spreading over a solid substrate under the action of various forces are known to exhibit a contact line instability. We use an example of thermally driven spreading on a horizontal surface to study how this instability can be suppressed, or patterns selected, using feedback control. We show that variation in the temperature imposed behind the contact line and proportional to the deviation of the contact line from its mean position produces a stabilizing effect on the dynamics of long wavelength unstable modes through local changes in the mobility of the liquid film. Theoretical results are supported by numerical simulations of the lubrication equations.

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I. INTRODUCTION

Driven spreading is a process which occurs in numerous industrial coating applications, from dip-coating and spin-coating to blow-off coating, so understanding its dynamics and learning to control it is very important. For instance, instabilities which arise during the spreading of the liquid on the solid substrate can lead to nonuniform coverage, adversely affecting the quality of produced coating. Driven spreading of thin films and patterning also have important implications for microfluidics.

Driven spreading of liquid films under the action of gravity [1], centrifugal acceleration [2], thermocapillary effects [3], or combination thereof [4] has been studied from both the linear [5] and nonlinear [6] perspective, and substantial progress has been reached in understanding the stability of the flow [7]. Considerable progress has also been reached in *active*, or feedback, control of *flat* liquid layers [8–10], whose dynamics is governed by normal differential operators. The attempts to influence the stability of spreading films have so far been limited to *passive*, or non-feedback, control achieved through either imposing an externally generated counterflow [11] or chemically patterning the substrate [12,13].

This study represents the first theoretical treatment of the active control problem for spreading films. The spatially and temporally nonuniform nature of spreading films makes the control problem much more difficult compared to the case of flat stationary films, because the dynamics of the former is governed by a nonnormal evolution operator and thus requires a completely different analysis. We derive the slip model of thermally driven spreading and use it to show that the contact line instability can be suppressed using adaptive temperature perturbations which depend on the distortion of the contact line. (This type of feedback is chosen because it is easiest to implement experimentally with sufficient spatial and temporal resolution via optical means [14].) Although the results of the following analysis should be applicable regardless of the driving force, we concentrate our attention on the case of thermal driving.

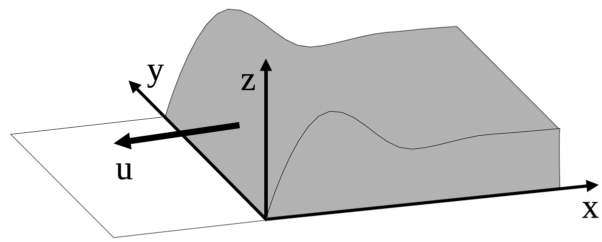

FIG. 1. Spreading liquid film on a solid substrate.

The layout of the paper is as follows. The slip model of thermally driven liquid films is derived in Section II and its stability analysis is conducted in Section III. Active control of the contact line instability is considered in Section IV. Numerical simulations of the model are presented in Section V and discussion of the results in Section VI.

II. SLIP MODEL FOR THERMAL SPREADING

We consider the spreading of a thin layer of partially wetting liquid on a horizontal substrate (see Fig. 1). The spreading process is conventionally described using the lubrication approximation [15], with the horizontal velocity governed by the Stokes equation

$$\mu\partial_{zz}\mathbf{v} = \nabla\bar{p}, \tag{1}$$

where μ is the dynamic viscosity, $\bar{p}$ is the modified pressure, and the vertical velocity is neglected.

It is well known [16] that the standard no-slip boundary condition at the substrate results in a stress singularity at the contact line. The only approach explored in the literature for the thermally driven case was to relieve this singularity by introducing a thin precursor film [4]. However, since precursors have never been observed experimentally, we pick a partial slip boundary condition introduced by Greenspan [17] for modeling the unforced spreading of liquid drops

$$\mathbf{v} = \frac{\alpha}{3\mu h} T \cdot \mathbf{n} \tag{2}$$

at the bottom and the stress balance boundary condition

$$T \cdot \mathbf{n} = \nabla\sigma - \kappa\sigma\mathbf{n} \tag{3}$$

at the free surface of the liquid layer, where T is the stress tensor, $\mathbf{n}$ is the unit vector in the z-direction, h is the local thickness of the film, α is the phenomenological slip coefficient, and σ is the surface tension coefficient. The curvature of the liquid-gas interface in the lubrication approximation is $\kappa = \nabla^2 h$. Solving (1) subject to these boundary conditions we obtain the horizontal velocity

$$\mathbf{v} = \frac{1}{\mu} z\nabla\sigma - \frac{1}{\mu}\left(\frac{\alpha}{3} + hz - \frac{1}{2}z^2\right)\nabla\bar{p}. \tag{4}$$

In order to make the phenomenological boundary condition (2) consistent with the physics of the flow, in (4) we have dropped an unphysical term $2\alpha\nabla\sigma/3\mu h$ which blows up for vanishing film thickness.

Assuming that the film is sufficiently thin, such that the effects of the hydrostatic pressure can be ignored, the modified pressure is given by the normal component of the surface tension $\bar{p} = -\kappa\sigma$. Substituting (4) into the mass conservation condition

$$\partial_t h = -\int_0^h (\nabla \cdot \mathbf{v}) dz \tag{5}$$

and integrating we obtain an evolution equation for the thickness:

$$\partial_t h = -\nabla\left[\frac{1}{2\mu}h^2\nabla\sigma + \frac{1}{3\mu}(\alpha h + h^3)\nabla(\sigma\nabla^2 h)\right]. \tag{6}$$

Now consider the situation which arises when the substrate covered by the liquid film is subjected to a linear temperature gradient in the x-direction. Assuming that the surface tension changes linearly with temperature θ,

$$\sigma(x) = \sigma(\theta_0) + x\partial_x\theta\partial_\theta\sigma \equiv \sigma_0 - \tau x, \tag{7}$$

and neglecting the variation in σ in the second term of (6), which produces subdominant contribution (see, e.g., the discussion in [4]), we obtain

$$\begin{aligned} \partial_t h = \frac{\tau}{2\mu}\partial_x h^2 - \frac{\sigma_0}{3\mu}\partial_x[(\alpha h + h^3)(\partial_{xxx}h + \partial_{xyy}h)] \\ - \frac{\sigma_0}{3\mu}\partial_y[(\alpha h + h^3)(\partial_{xxy}h + \partial_{yyy}h)]. \end{aligned} \tag{8}$$

We can absorb all parameters into the spatial and temporal scales by introducing the nondimensional variables $t' = t/T$, $x' = x/X$, $y' = y/X$, and $h' = h/H$. Setting $H^2 = \alpha$, $X^3 = 2\sigma_0\alpha/3\tau$, $T = 2\mu X/\tau H$, and dropping the primes we can rewrite (8) as

$$\begin{aligned} \partial_t h = \partial_x h^2 - \partial_x[(h + h^3)(\partial_{xxx}h + \partial_{xyy}h)] \\ - \partial_y[(h + h^3)(\partial_{xxy}h + \partial_{yyy}h)]. \end{aligned} \tag{9}$$

It is worth mentioning that (9) has the same form as an equation describing gravity driven rather than temperature driven films (see, e.g., equation (33) in [18]), with the exception that h in the first term on the right-hand-side is replaced with h^2. This suggests that the gravity driven case can be treated in essentially the same way.

The liquid spreads in the direction opposite to the temperature gradient, so the motion of the contact line is most conveniently described in the reference frame moving with speed u towards negative x. In this frame the equations possess a transversely uniform steady state solution, which gives the asymptotic film profile for constant flux boundary conditions. Substituting $h(x, y, t) = h_0(x + ut)$ into (9) and integrating once we obtain

$$uh_0 - h_0^2 + (h_0 + h_0^3)h_0''' = d. \tag{10}$$

The constants u and d can be determined from the appropriate boundary conditions. For instance, at the contact line, $x = 0$, the thickness has to vanish, $h_0(0) = 0$. Furthermore, if the dynamic contact angle in the dimensional variables is γ, we also have $h_0'(0) = c$ with $c \equiv (X/H)\tan\gamma$. The constant flux boundary conditions far away from the contact line can be written as $h_0(\infty) = h_\infty$ and $h_0'''(\infty) = 0$, such that $u = h_\infty$ and $d = 0$, and consequently

$$h_0''' = \frac{h_0 - h_\infty}{h_0^2 + 1}. \tag{11}$$

The solution of this equation describes the height profile of the spreading film once the distance from the contact line to the reservoir becomes sufficiently large.

III. CONTACT LINE INSTABILITY

Linear stability of the asymptotic solution h_0 can be determined in a standard way. Since this solution is uniform in the transverse direction, the linearized equation can be partially diagonalized by Fourier transforming it in the y-direction. By substituting

$$h(x, y, t) = h_0(x + ut) + \epsilon g(x + ut, t)e^{iqy} \tag{12}$$

into (9) and retaining terms of order ϵ, we obtain an equation describing the dynamics of small perturbations:

$$\partial_t g = L(q)\, g, \tag{13}$$

where $L(q) \equiv L_0 + q^2 L_1 + q^4 L_2$ is a fourth order differential operator defined via

$$\begin{aligned} L_0\, g &= -[\{h_\infty - 2h_0 + (1 + 3h_0^2)h_0'''\}g + (h_0 + h_0^3)g''']', \\ L_1\, g &= [(1 + 3h_0^2)h_0' g]' + 2(h_0 + h_0^3)g'', \\ L_2\, g &= -(h_0 + h_0^3)g. \end{aligned} \tag{14}$$

Even though we cannot find the eigenfunctions and eigenvalues of $L(q)$ for arbitrary q analytically, for long wavelength disturbances we can use regular perturbation theory to get the leading order (in q^2) terms. This requires finding the eigenfunctions of L_0 and its adjoint, $L_0^\dagger$.

Taking the second derivative of (10) we obtain

$$L_0\, h_0' = 0, \tag{15}$$

so that $g_0 = h_0'$ is an eigenfunction of L_0 with eigenvalue $\lambda_0^0 = 0$. The adjoint operator is found to be

$$\begin{aligned} L_0^\dagger f = {} & [h_\infty - 2h_0 + (1 + 3h_0^2)h_0'''] f' \\ & - [(h_0 + h_0^3) f']''', \end{aligned} \tag{16}$$

so its respective eigenfunction is just a constant, say, $f_0 = 1$. In fact, this is a very generic result with deep physical meaning. Identical relations between the asymptotic state and the leading eigenfunctions were obtained, e.g., for gravity driven films using the precursor model [6,7]. The relation for g_0 is due to the fact that equations for the asymptotic state are translationally invariant in the direction of the flow (this reflects an arbitrary choice in the position of the contact line), while the relation for f_0 is the consequence of the gradient form of (5), which reflects mass conservation.

As a comparison of (14) and (16) shows, the operator L_0 is nonnormal, and therefore the validity of modal analysis is questionable due to a possibility of transients [7]. For now we will assume that the modal analysis is valid and continue, delaying the discussion of the effects of nonnormality to section VI.

The perturbation theory dictates the following q-dependence of the leading eigenvalue:

$$\lambda_0(q) = \lambda_0^0 + q^2 \frac{\int_0^\infty f_0\, L_1\, g_0 dx}{\int_0^\infty f_0 g_0 dx} + O(q^4). \tag{17}$$

Using (11) this can be reduced to

$$\lambda_0(q) = \frac{q^2}{h_\infty} \int_0^\infty h_0(h_0 - h_\infty) dx + O(q^4). \tag{18}$$

This eigenvalue determines the growth rate of the disturbance with the spatial structure given by the leading eigenfunction. It is easy to see that, if the asymptotic profile is monotonic, $0 < h_0 < h_\infty$, the integral is strictly negative and the system is stable with respect to long wavelength disturbances (the term of order q^4 is negative, as $\lambda_0(q) \to -(h_\infty + h_\infty^3) q^4$ for $q^2 \to \infty$). However, a capillary ridge near the contact line can make the integral positive, showing that the increased mobility of the ridge provides the mechanism for the long wavelength instability in the thermally driven case. This mechanism has been originally conjectured by Kataoka and Troian based on the energy analysis of the precursor model [4], but never proved. The result (18) proves this conjecture, in addition giving an explicit condition on the shape of the capillary ridge, and echoes a similar result obtained for the case of gravity-driven flows [7].

Substituting $g(x,t) = h_0'(x) \exp(\lambda_0(q) t)$ into (12) we notice that for small disturbances the right hand side represents the first two terms of the Taylor expansion of $h_0(x + \xi + ut)$, where

$$\xi(y,t) = \epsilon e^{iqy + \lambda_0(q) t}. \tag{19}$$

is the deviation of the contact line from the mean. In fact, the marginal translational mode $g_0 = h_0'$ is not the only eigenfunction of L_0. There is a discrete spectrum of eigenvalues λ_n and eigenfunctions g_n. Therefore, in the presence of an arbitrary disturbance (19) will read

$$\xi(y,t) = \frac{1}{c} \sum_n g_n(0) \int_{-\infty}^\infty \epsilon(q) e^{iqy + \lambda_n(q) t} dq. \tag{20}$$

In the unstable regime the amplitude of the distortion will grow exponentially fast and eventually the contact line will take the form of “fingers” or rivulets [19].

IV. ACTIVE CONTROL OF THE CONTACT LINE INSTABILITY

Can we suppress the contact line instability, or alternatively, can we impose a pattern of a desired wavelength? In principle, the answer seems to be clear, as ways of controlling the dynamics were suggested for other fluid systems involving liquid layers, where the instability is produced by buoyancy [8], thermocapillary effects [9], or evaporation [10]. So it would seem that the control methods developed for those other systems could be easily adapted for controlling driven spreading. In reality the spreading films turn out to be dramatically different.

All existing control methods can only be applied for stabilizing *uniform* target states, i.e., flat films. As a consequence of uniformity, the differential operators describing the dynamics of disturbances commute with the translation operator, and hence, their horizontal components completely diagonalize in the Fourier space, which has far reaching implications. First of all, each mode can be controlled completely independently of the others. Second, such differential operators are symmetric, so the effect of any perturbation on any mode can be calculated without knowledge of any other modes. Finally, being symmetric, such operators are also normal, so there are no transients and the modal analysis is unconditionally valid.

The target state $h_0(x,t)$ in the present problem is *nonuniform* in the direction of the flow. As a result, the differential operator L does not fully diagonalize and the control problem becomes vastly more complicated. Feedback applied to one mode generally affects all others, so an infinite-dimensional problem has to be considered from the outset. It is impossible to calculate the effect of feedback on the dynamics of any of the modes without the full knowledge of all left eigenfunctions f_n. And even having overcome these daunting problems, making the dynamics asymptotically stable does not guarantee that that the transient effects will not invalidate the whole analysis.

To see if it is possible to make any progress in controlling the contact line instability, let us restrict our attention to monochromatic disturbances $\epsilon g(x+ut, t) \exp(iqy)$

for the moment. Since the flow is driven by the gradient in the temperature (and hence surface tension), the stability of the flow is most easily altered by varying the temperature field behind the contact line. Suppose we modify the temperature profile by adding a perturbation

$$\Delta\theta(x,y,t) = -\epsilon\tau(\partial_\theta\sigma)^{-1}s(t)w(x+ut)e^{iqy}, \qquad (21)$$

where the transverse wavelength q is the same as that of the disturbance, and s and w are some arbitrary functions. Consequently, (7) and hence (13) will be modified to account for the variation in the surface tension transversely as well as along the direction of the flow. At order ϵ (neglecting terms of order q^4 and higher) we obtain

$$\partial_t g = L_0\, g + q^2 L_1\, g + s\left[(h_0^2 w')' - q^2 h_0^2 w)\right]. \qquad (22)$$

To get a sense of the dynamics of different modes, we expand g in the basis formed by the eigenfunctions g_n,

$$g(x,t) = \sum_m G_m(t) g_m(x), \qquad (23)$$

and make the strength of the applied perturbation proportional to the magnitude of the distortion of the contact line (with a constant k to be determined later)

$$s(t) = k\frac{g(0,t)}{c} = \frac{k}{c}\sum_m G_m(t) g_m(0). \qquad (24)$$

Finally, (22) is reduced to an infinite system of ODEs by multiplying it by f_n and integrating from 0 to ∞:

$$\dot{G}_n = \lambda_n^0 G_n + \sum_m A_{nm} G_m + q^2 \sum_m B_{nm} G_m, \qquad (25)$$

where

$$\begin{aligned} A_{nm} &= \frac{k g_m(0)}{c h_\infty}\int_0^\infty f_n (h_0^2 w')' dx, \\ B_{nm} &= \frac{1}{h_\infty}\int_0^\infty f_n\left(L_1\, g_m - \frac{k}{c} g_m(0) h_0^2 w\right) dx. \end{aligned} \qquad (26)$$

This system cannot be solved because we do not know the eigenfunctions f_n and g_n for $n \geq 1$. However, assuming that the eigenvalues λ_n^0 are all negative and have a sufficiently large magnitude (surface tension dominates for short wavelength disturbances, so $\lambda_n^0 = \lambda_n(0) \to -\infty$ for $n \to \infty$, and for each n, $\lambda_n(q) \to -\infty$ for $q^2 \to \infty$), we can truncate the system only taking into account the dynamics of the leading mode $n = 0$, such that

$$\dot{G}_0 = (A_{00} + q^2 B_{00}) G_0. \qquad (27)$$

For the solution $G_0 = 0$ to be stable we need

$$\begin{aligned} A_{00} &= k h_\infty w'(\infty) \leq 0, \\ B_{00} &= \frac{1}{h_\infty}\int_0^\infty \left[h_0(h_0 - h_\infty) - k h_0^2 w\right] dx \leq 0, \end{aligned} \qquad (28)$$

The first condition is automatically satisfied for any $w(x)$ with finite support, because in this case $A_{00} = 0$, while the second conditions can always be satisfied with the proper choice of the gain k. Since the choice of k is independent of q, we can immediately generalize to non-monochromatic disturbances by integrating over all q, such that the feedback will be given by

$$\Delta\theta(x,y,t) = -k\tau(\partial_\theta\sigma)^{-1} w(x+ut)\xi(y,t), \qquad (29)$$

where $\xi(y,t)$ is the instantaneous deviation of the contact line from its mean position.

The action of this feedback is easy to interpret: the instability is suppressed by exploiting the very mechanism that produces it, i.e., by changing the mobility of the film. By heating the film behind the advanced regions of the contact line, while simultaneously cooling the film behind the retarded regions, the liquid is redistributed in such a way as to decrease the thickness, and hence the mobility, where we need to slow down the motion of the contact line and increase the thickness and mobility, where we need to speed it up to compensate for the deviation. Furthermore, even though we used the long wavelength limit to determine stability, we should expect that short wavelength disturbances will also be stabilized, because the suggested control method achieves control by quenching the basic destabilization mechanism.

V. NUMERICAL SIMULATIONS

We have made several assumptions (validity of modal analysis, long wavelength approximation, truncation of stable modes) during the derivation of the control law, any one of which could in principle invalidate the final result. Even though most seem very plausible, the only way to prove that the proposed feedback control is effective is to do experiments or perform numerical simulations of the governing equations. While the theory has been confirmed by experiments [20], let us present the results of our numerical simulations here.

Within the slip model the dynamics of the film is governed by (6), where in the presence of feedback (29) the surface tension is given by

$$\sigma(x,y,t) = \sigma_0 - \tau x + k\tau w(x+ut)\xi(y,t). \qquad (30)$$

From the numerical standpoint it is more convenient to perform all calculations in the frame moving with the velocity u in the direction of the flow, in which the base state is stationary, so upon nondimensionalization (9) will be replaced with

$$\begin{aligned} \partial_t h = \partial_x[h^2 - uh] &- \partial_x[(h+h^3)(\partial_{xxx}h + \partial_{xyy}h)] \\ &- \partial_y[(h+h^3)(\partial_{xxy}h + \partial_{yyy}h)] \\ &- k\partial_x[h^2\partial_x w]\xi - k\partial_y[h^2\partial_y\xi]w. \end{aligned} \qquad (31)$$

This equation is discretized using second order correct finite difference approximations of spatial derivatives on a rectangular mesh. Periodic boundary conditions are used in the y-direction. The boundary conditions on the contact line are

$$\begin{aligned} c &= [1 + (\partial_y \xi)^2]^{1/2} \partial_x h, \\ h_\infty &= \partial_y \xi (\partial_{xxy} h + \partial_{yyy} h) - (\partial_{xxx} h + \partial_{xyy} h), \end{aligned} \qquad (32)$$

where the mean position $x_0(t)$ and the zero mean distortion $\xi(y,t)$ of the contact line are obtained by solving $h(x_0 + \xi, y, t) = 0$. Finally, the boundary conditions at the tail of the film are $h = h_\infty$ and $\partial_{xxx} h = 0$.

The time derivative is discretized to obtain an implicit Euler scheme. The resulting system of nonlinear equations

$$h_{ij}(t + dt) = h_{ij}(t) + L_{ij}[h(t + dt)]dt \qquad (33)$$

is solved using Newton's iterations to obtain the height profile at the next step. Automatic time step control is implemented by comparing the difference between the result $h_1 = h(t+dt)$ after one full time step and the result $h_2 = h(t + \frac{dt}{2} + \frac{dt}{2})$ after two successive half-steps with a predefined threshold. Finally, a second order correct in time iteration scheme is obtained by performing a locally extrapolated step doubling:

$$h(t + dt) = 2h_2 - h_1. \qquad (34)$$

The position of the contact line is re-calculated at each step and the boundary conditions at the contact line are updated accordingly.

The simulations were conducted for a particular choice of parameters ($h_\infty = c = 1$) on a rectangular domain of size $l_x = 40$ in the direction of the flow and $l_y = 40$ or $l_y = 80$ in the transverse direction, covered by a uniform mesh of size 400×400. The initial condition was chosen as the base state profile shifted in the x-direction according to a weakly perturbed position of the contact line,

$$h(x, y, 0) = h_0(x - x_0 - \xi(y, 0), y). \qquad (35)$$

The state was then allowed to evolve without control, $k = 0$, until the magnitude of the distortion $s(t) = \langle \xi^2(y,t) \rangle_y^{1/2}$ exceeded a threshold value of $s_{max} = 0.25$ at some $t = t_0$, after which the feedback gain was gradually increased to the maximal value $k = 1$ according to the following law:

$$k(t) = 1 - e^{t_0 - t}. \qquad (36)$$

The spatial profile of the temperature perturbation in the direction of the flow was chosen to be parabolic,

$$w(x, t) = \frac{4}{w_0}(x - x_0)(w_0 - x + x_0), \qquad (37)$$

with approximately the same width $w_0 = 5$ as the capillary ridge. This arrangement corresponds to applying the perturbations right under the ridge, where the effect of the feedback is maximal.

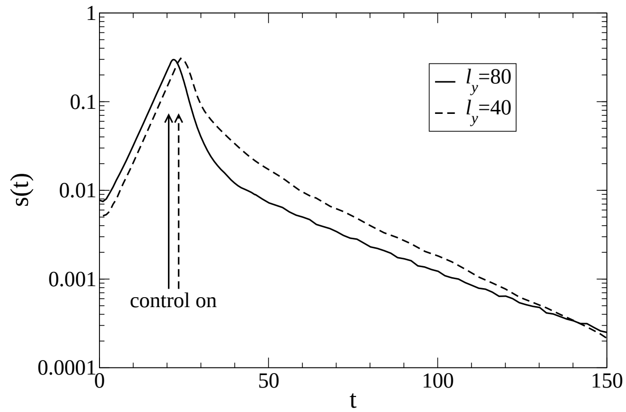

FIG. 2. Magnitude of the distortion of the interface, $s(t)$, for films of lateral size $l_y = 40$ and $l_y = 80$. The arrows indicate when control is turned on.

As Fig. 2 shows, before the feedback is turned on, the magnitude of the distortion of the contact line grows exponentially fast, with an exponent determined by the most unstable eigenmode, $q = q_{max}$, independent of the lateral size of the system. The instability is driven by the excess of liquid in the capillary ridge behind the advanced parts of the interface (see Fig. 3(a)). After the feedback is turned on, following a short initial transient period during which the liquid under the capillary ridge is redistributed in such a way that the thickness is reduced behind the advanced parts of the film in favor of the regions behind the retarded parts of the film (see Fig. 3(b)), the distortion starts to decrease. Initially the decay rate is quite large, as disturbances with $q \approx q_{max}$ (which have grown the most during the uncontrolled period of evolution) are quickly suppressed. After this the the asymptotic regime, characterized by exponential decay with a much smaller rate, sets in. Indeed, (27) and (28) show that our control is less effective in suppressing disturbances with smaller wavenumbers as the liquid has to be redistributed over larger distances. The asymptotic decay rate is determined by the smallest wavenumber disturbance present in the system, $q_{min} = 2\pi/l_y$, and therefore has to depend on the lateral size of the system, which is confirmed by Fig. 2. All these features are consistent with the previously presented theory.

VI. DISCUSSION

The above analysis was conducted in the assumption that the modal analysis of the linearized equation (13) gives an accurate representation of *short term* dynamics, which is not obvious *a priori*, since the differential operator L_0 (and hence L) is nonnormal. As discussed by Bertozzi and Brenner [7], even if the dynamics is *asymptotically* stable, small perturbations at the contact line may lead to transients which could be amplified by orders of magnitude due to the close alignment of some of

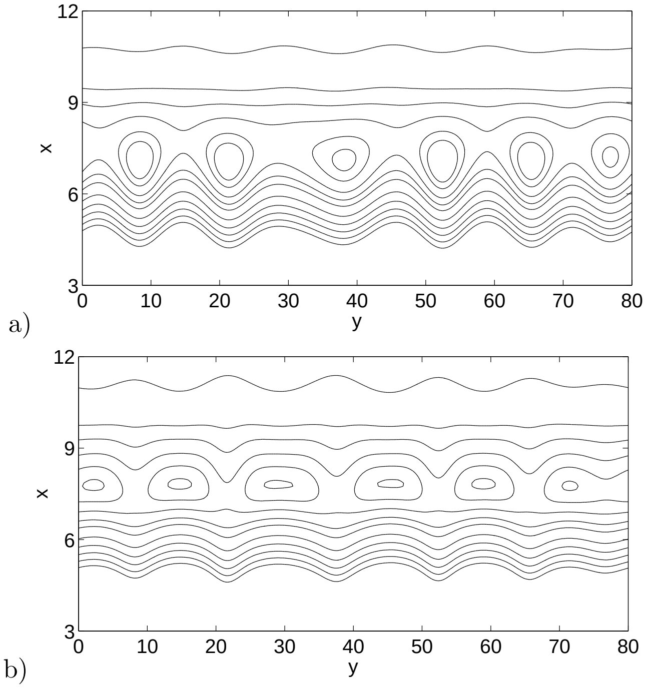

FIG. 3. Film profile (a) at $t = 20$ (before control is turned on) and (b) at $t = 25$ (after the feedback gain asymptotes). The liquid is flowing down and the bottom contour shows the position of the contact line.

the eigenfunctions. However, the analysis of, and hence the conclusions drawn for, the precursor model of gravity driven spreading [6,7,21] are not directly applicable in our case. On the contrary, the existing experimental data [3,4,19], agrees with the predictions of the linear theory rather well, suggesting that the nonnormality is weak, and therefore the modal analysis used here accurately describes both the short and long term dynamics. The results of direct numerical simulations of the lubrication equations provide additional support for the validity of the presented analysis.

In principle, feedback control can be made effective even for strongly nonnormal systems, such as the gravity driven spreading at certain inclination angles. Indeed, the amplification due to nonnormality strongly depends on the timescale of the transient. This timescale can be reduced substantially by converting the weakly stable modes into strongly stable ones. However, because small disturbances at the contact line could be transiently amplified to produce $O(1)$ changes in the thickness of the capillary ridge, it is possible that the control algorithm will have to use direct measurements of the thickness rather than the much easier to monitor position of the contact line.

Certain care also has to be used in interpreting the results of stability analysis in the presence of feedback. In general, one has no right to truncate the system (25), even though all the modes with $n \geq 1$ are stable without feedback. It is easy to see that, since neither A_{n0} nor B_{n0} vanish, the feedback designed to suppress the 0th mode affects not just that particular mode, but the other modes as well, so potentially it could lead to destabilization of the formerly stable modes. However, if the modes with $n = 1, 2, \cdots$ are sufficiently well damped, their stability will not be affected by control spillover from mode $n = 0$. This turns out to be the case, as direct numerical simulations and experiments [20] show. Moreover, the degree to which the feedback affects different modes can be modified by changing the profile of the perturbation $w(x)$, so the spillover effect can be controlled. However, as the integrals (26) show, the detailed information about the shape of the eigenfunctions $f_n(x)$ is necessary in order to make more quantitative predictions. For instance, the effect of feedback on the leading mode is greatest when the perturbation is concentrated under the capillary ridge, where h_0^2 is at its maximum.

The approach presented in this paper offers significant advantages in controlling the dynamics of microflows compared to the one based on chemical patterning of the substrate [12,13]. On the one hand no preparation of the substrate is needed, while the patterns can be dynamically reconfigured offering potential for a significant increase in flexibility. For instance, once the instability is suppressed, selective patterning can be achieved by removing feedback at the desired wavelength q_0, i.e., making the gain q-dependent, $k(q) = k_0(1 - \delta(|q| - q_0))$. On the other hand, feedback control can be used to achieve extremely small feature size, if high intensity radiation is used to drive the flow on a thin substrate with small conductivity [10], opening up new prospects for microfluidics and microfabrication applications.

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