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Georgia Institute of Technology
School of Physics

Solutions to the Final Exam (prepared by Debin Wang)

PHYS 2211 L (Intro to Physics)
Instructor: Slaven Peles

Duration: 160 minutes

Standard calculators allowed

This test form consists of **9** pages and **8** questions. Please bring any discrepancy to the attention of a proctor. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

- Print your name, Georgia Tech ID number, and sign the test form.
- In each problem first define all quantities (e.g. v_{01} is initial velocity of the first object, a is the acceleration, ...) before proceeding with your calculations.
- Show all the steps of your calculation and provide explanations when necessary. If you need more space continue working on the back of the test form sheet.
- Explain the physical meaning of your results.

For graders' use:

	Score		Score		Score		Score
1 (25)		3 (25)		5 (25)		7 (25)	
2 (25)		4 (25)		6 (25)		8 (25)	
Subtotal		Subtotal		Subtotal		Subtotal	

Total (200)	
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1. [25] A particle is constrained to move in a circle with a 10-meter radius. At one instant, the particle's speed is 10 meters per second and is increasing at a rate of 10 meters per second squared. Find the angle between the particle's velocity and acceleration vectors.

Solution:

For the motion of this particle, its acceleration can be decomposed into two orthogonal parts a_n and a_t :

$$\begin{aligned} a_n &= \frac{v^2}{R} \\ a_t &= \dot{v} \end{aligned} \tag{1}$$

The angle between the particles velocity and acceleration θ satisfies:

$$\tan \theta = \frac{a_n}{a_t} = \frac{\frac{v^2}{R}}{\dot{v}} = \frac{(10m/s)^2}{10m/s^2} = 1 \Rightarrow \theta = 45^\circ \tag{2}$$

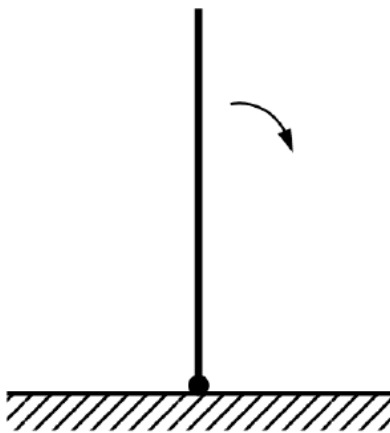


Figure 1.

2. [25] A thin uniform rod of mass M and length L is positioned vertically above an anchored frictionless pivot point, as shown above, and then allowed to fall to the ground. With what speed does the free end of the rod strike the ground?

Solution:

Before the rod begins to fall, the initial total energy is

$$E_i = Mg\frac{L}{2}$$

At the instant the rod falls onto the ground, the total energy equals its kinetic energy

$$E_f = \frac{1}{2}I\omega^2, \quad \text{where } I = \frac{1}{3}ML^2 \quad (3)$$

Since the only force acting on the system is gravity, which is a conservative force, the energy is conserved and $E_i = E_f$. From there we find

$$\omega = \sqrt{\frac{3g}{L}} \Rightarrow v = \omega L = \sqrt{3gL}$$

Note that kinetic energy of the rod when it hits the ground can be also calculated as

$$K_f = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2 = \frac{1}{2}\left(\frac{ML^2}{4} + \frac{ML^2}{12}\right)\omega^2 = \frac{1}{2}I\omega^2 \quad (4)$$

what gives you the same result as in (3).

3. [25] A ball of radius 12 cm and mass 5.0 kg is falling through the air at its terminal velocity. How much energy is dissipated during 5 seconds? Assume that the drag force is well approximated by the expression $\vec{D} = kAv^2\hat{v}$, where v is velocity, and A is the cross section of the object moving through a resistive environment. For the air constant $k = 0.25 \text{ kg/m}^3$.

Solution:

While the ball is falling through the air at its terminal velocity, its kinetic energy conserves:

$$\begin{aligned}\Delta KE &= W_g + W_f = 0 \\ &\Rightarrow W_f = -W_g \\ \Rightarrow Q_d &= -W_f = W_g = mg\Delta h\end{aligned}\tag{5}$$

i.e., the magnitude of the dissipated energy Q_d equals to the work done by gravity For Δh :

$$\begin{aligned}\Delta h &= vt \\ &= \sqrt{\frac{mg}{kA}}t\end{aligned}\tag{6}$$

Hence,

$$\begin{aligned}\Delta h &= vt \\ Q_d &= mg\Delta h \\ &= (mg)^{\frac{3}{2}}(k\pi R^2)^{-\frac{1}{2}}t \\ &= 1.61 \times 10^4 J\end{aligned}\tag{7}$$

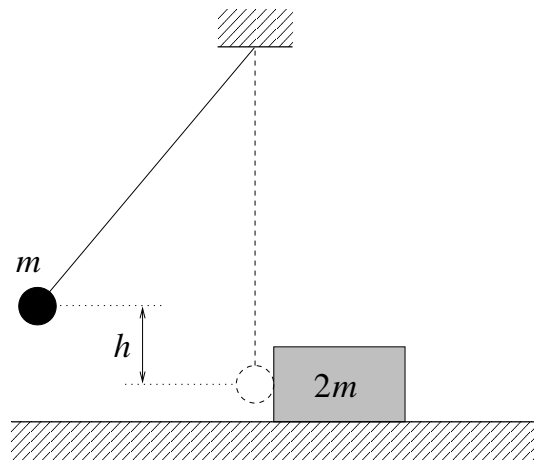


Figure 2.

4. [25] A ball of mass m suspended on the end of a wire is released from height h and collides elastically with a block of mass $2m$ at rest on a frictionless surface. What is the maximum height that the ball reaches after the collision?

Solution:

Use V as the block's velocity, and m, v_i, v_f as the ball's mass, velocity before and after the collision, respectively. The collision is elastic, so the total momentum and the total energy are conserved:

$$\begin{aligned} mv_i &= mv_f + 2mV \\ \frac{1}{2}m(v_i)^2 &= \frac{1}{2}m(v_f)^2 + \frac{1}{2}2mV^2 \end{aligned} \tag{8}$$

By eliminating V from the equations above we obtain

$$(v_i)^2 = (v_f)^2 + \frac{(v_i - v_f)^2}{2},$$

and thus $v_f = -v_i/3$. The relation between maximal height and initial velocity is obtained from the energy conservation

$$mgh = \frac{1}{2}mv^2. \tag{9}$$

Hence, we have the relation between the initial and final height

$$h_f = \frac{(v_f)^2}{(v_i)^2} h_i = \frac{1}{9}h \tag{10}$$

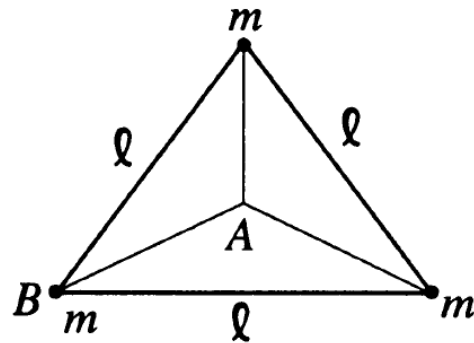


Figure 3.

5. [25] Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown in Figure 3. The system rotates with constant angular velocity ω around an axis normal to the triangle. Find the ratio of the kinetic energy of the system if it rotates about an axis through point B versus an axis through point A (center of mass for the system).

Solution:

For this system, total kinetic energy:

$$K = \sum_i \frac{1}{2} I_i \omega^2$$

For rotation about point A:

$$I_1 = I_2 = I_3 = m \frac{l^2}{3}$$

For rotation about point B:

$$I_1 = I_2 = ml^2, I_3 = 0$$

Hence, we have:

$$\frac{K_B}{K_A} = \frac{\frac{1}{2} m \omega^2 \times l^2 \times 2}{\frac{1}{2} m \omega^2 \times \frac{l^2}{3} \times 3} = 2$$

(11)

6. [25] Ann (mass 50 kg) is standing at the left end of a 15-m-long, 500 kg cart that has frictionless wheels and rolls on a frictionless track. Initially both Ann and the cart are at rest. Suddenly, Ann starts running along the cart at a speed of 5.0 m/s relative to the cart. How far will Ann have run relative to the ground before she reaches the right end of the cart?

Solution:

Let us denote mass of the cart with M , length of the cart with L , and Ann's mass with m . Furthermore, let v_M be the velocity of the cart, v_m Ann's velocity in the ground frame, and v'_m Ann's velocity in the cart frame of reference. Since initially the total momentum of Ann and the cart is zero, from momentum conservation we obtain:

$$Mv_M + mv_m = 0 \quad (12)$$

From the other hand Galilean transformations give us:

$$v_m - v_M = v'_m \quad (13)$$

After eliminating v_M from equations (12) and (13) we obtain

$$v_m = \frac{M}{M+m} v'_m$$

It takes an $\Delta t = L/v'_m$ time to run from one to the other end of the cart. Hence, relative to the ground, the distance Ann has run before reach the right end of the cart:

$$s = v_m \Delta t = \frac{M}{M+m} v'_m \frac{L}{v'_m} = \frac{ML}{M+m}$$

$$s = 13.64m$$

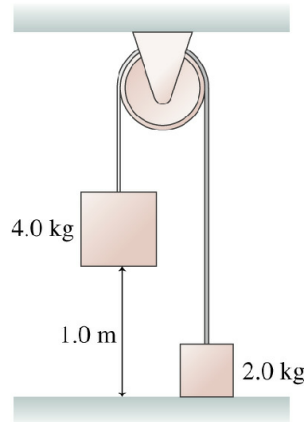


Figure 4.

7. [25] Two blocks are connected by a massless rope that passes over a pulley, as shown in Figure 4. The mass of the pulley is 1.2 kg and its radius is 5.0 cm. If the blocks are released from rest how long does it take the 4 kg block to reach the floor? Assume there is no dissipation in the system, and the rope does not slide over the pulley.

Solution:

Denote $m_1 = 4$ kg, $m_2 = 2$ kg, gravity acceleration $g = 9.8$ m/s², mass of the pulley $M = 1.2$ kg, and the radius of the pulley $R = 5.0$ cm.

By using Newton's Second and Third Law, and considering the constraints in the system we can obtain the equation of motion

$$(m_1 + m_2 + I/R^2)a = (m_1 - m_2)g, \quad (14)$$

and from there find the acceleration

$$a = \frac{m_1 - m_2}{m_1 + m_2 + M/2} g = 2.72 \text{ m/s}^2. \quad (15)$$

Here we approximated pulley with a disk so the moment for the pulley is $I = MR^2/2$.

Since the acceleration of the system is constant, we can find the time it takes the heavier block to hit the ground from the expression

$$h = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{a}} = 0.857 \text{ s}$$

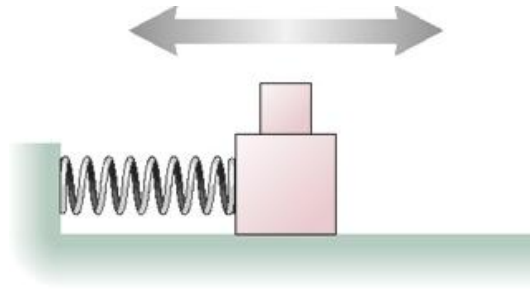


Figure 5.

8. [25] The Figure 5 shows a 1.0 kg mass riding on top of a 5.0 kg mass as it oscillates on a frictionless surface. The spring constant is 50 N/m and the coefficient of static friction between the two blocks is 0.50. What is the maximum oscillation amplitude for which the upper block does not slip?

Solution:

The net force acting on the upper block m_1 is the force of friction due to the lower block m_2 . The model of static friction gives the maximum force of static friction as

$$f_{s \max} = \mu_s m_1 g = m_1 a_{\max} \Rightarrow a_{\max} = \mu_s g.$$

Using $\mu_s = 0.5$, $a_{\max} = \mu_s g = 4.9 \text{ m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{\max} . We can calculate the maximum oscillation amplitude A as follows:

$$a_{\max} = \omega^2 A = \frac{k}{m_1 + m_2} A \Rightarrow A = \frac{m_1 + m_2}{k} a_{\max} = 0.588 \text{ m}$$

End of examination

Total pages: 9

Total marks: 200