Given name:____________________ **Family name:**____________________

GTID:____________________ **Signature:**__________________________

Georgia Institute of Technology
School of Physics

Test Form 101L

PHYS 2211 (Intro to Physics)
Instructor: Slaven Peleš

Duration: 80 minutes

Standard calculators allowed

This examination paper consists of **9** pages and **5** questions. Please bring any discrepancy to the attention of a proctor. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

1. Print your name, Georgia Tech ID number, and sign the test form.
2. In each problem first define all quantities (e.g. v_{01} is initial velocity of the first object, a is the acceleration, ...) before proceeding with your calculations.
3. Show all the steps of your calculation and provide explanations when necessary. If you need more space continue working on the back of the test form sheet.
4. Explain physical meaning of your results.

Scores will be posted on your class website. Quiz grades become final when the next test is given.

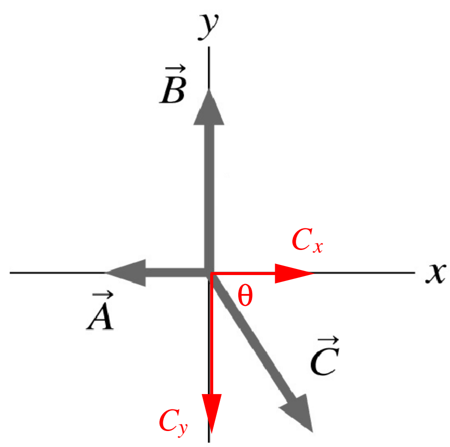

Figure 1.

1. [25] Three vectors shown in Figure 1 add to $1.0\hat{\boldsymbol{\jmath}}$. The magnitude of vector **A** is 1.0 and the magnitude of vector **B** is 3.0. What are the magnitude and the angular direction (with respect to x-axis) of vector **C**? By convention $\hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$ are unit vectors along x- and y-axis, respectively.

   Solution: From the Figure 1 we find that

$$\begin{aligned} \mathbf{A} &= -1.0\hat{\boldsymbol{\imath}} \\ \mathbf{B} &= 3.0\hat{\boldsymbol{\jmath}} \end{aligned}$$

   Vectors **A**, **B** and **C** add up to zero, therefore $\mathbf{C} = -\mathbf{A} - \mathbf{B}$. Writen in vector component form this equation becomes

$$\begin{aligned} C_x &= 1.0 \\ C_y &= -3.0 \end{aligned}$$

   In Euclidian space the norm of the vector is given by

$$|\mathbf{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

   The angle θ, which desrcibes orientation of the vector **C**, is

$$\theta = \arctan\left(\frac{|C_y|}{|C_x|}\right) = 63^\circ$$

   in clockwise direction with respect to the x-axis. Therefore, orientation of **C** is -63° or 297°.

2. [25] An object moving on the x-axis has the velocity as shown in Figure 2. Draw position and acceleration graphs for the object from $t = 0$s to $t = 5$s. Label axis tics appropriately, and draw additional tics if necessary. Choose the origin of the position diagram so that the initial position is zero.

   Solution: The velocity shown represents the slope (first time derivative) of the position curve to be sketched. The acceleration, in turn, is the slope of the velocity curve. The velocity is constant at 2 m/s for the first 2 s, so the position increases linearly (with slope 2 m/s) from (0 s, 0 m) to (2 s, 4 m). The acceleration during this period is 0 m/s^2.

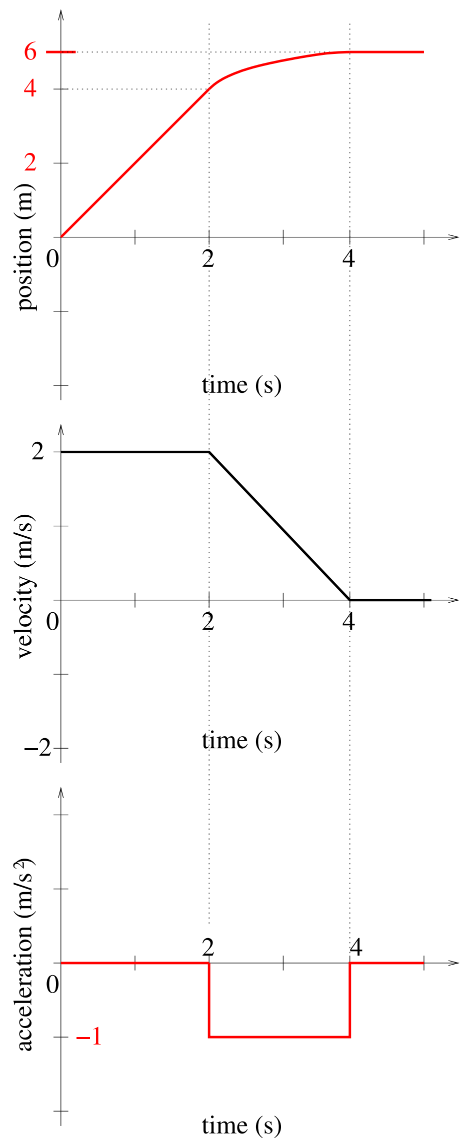

Figure 2.

The velocity then decreases linearly at -1 m/s^2 for 2 s. A quick calculation of the area under the curve tells us that the position function must pass through the points (3 s, 5.5 m) and (4 s, 6 m). Note that this segment (2 s $< t <$ 4 s) of the position curve is parabolic. The acceleration on this interval is constant at -1 m/s^2. From $t = 4$ s on, the velocity is 0 m/s, so the object does not move and the acceleration returns to 0 m/s^2.

3. [25] An object moving on the y-axis has a position described by $y(t) = -3t^3 + t^2 + 6$ where y is in meters when t is in seconds. Find (a) average acceleration between time $t = 0$s and time $t = 2.0$s; (b) instantenous acceleration at time $t = 3.0$s; and (c) discuss your results. Does the object slow down or speed up at $t = 3.0$s?

Solution:

(a) From the expression for the position of the object $y(t) = -3t^3 + t^2 + 6$ we find velocity as

$$v(t) \equiv \frac{dy}{dt} = -9t^2 + 2t, \tag{1}$$

where v is given in meters per second when the time t is expressed in seconds. The average acceleration is defined as

$$a_{avg} \equiv \frac{v(t_2) - v(t_1)}{t_2 - t_1},$$

so we find

$$a_{avg} = \frac{v(2) - v(0)}{2\text{ s} - 0\text{ s}} = \frac{(-9(2)^2 + 2(2) - 0)\text{ m/s}}{2\text{ s}} = -16\text{ m/s}^2$$

(b) Instantenous acceleration is a second time derivative of the position, so we get

$$a(t) \equiv \frac{d^2y}{dt^2} = -18t + 2 \tag{2}$$

where a is in meters per second square if t is expressed in seconds. At time $t = 3$s we find

$$a(3) = (-18(3) + 2)\text{ m/s}^2 = -52\text{ m/s}^2$$

(c) From the expression (2) we find that the acceleration a is positive for $t < 1/9$ s, and negative otherwise. Similarly, from the expression (1) we find the velocity v is positive within the time interval $0 < t < 2/9$ s, and negative elsewhere. Both, velocity and acceleration are negative for all $t > 2/9$ s. Therefore, at $t = 3$ s, a and v are in the same direction and the object is speeding up. The magnitude of the acceleration is also increasing.

4. [25] Heather and Jerry are standing on a bridge 10 m above a river. Heather throws a rock straight down with a speed of 3 m/s. Jerry, at exactly the same instant of time, throws a rock straight up with the same speed. Assume air resistance is negligible, and acceleration due to gravity is 9.8 m/s^2. Find (a) how much time elapses between the first splash and the second splash; and (b) what are respective velocities of the two rocks as they hit the water. (c) Discuss your results.

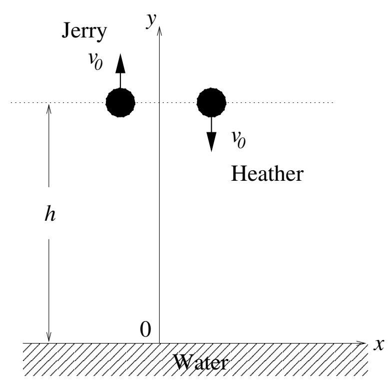

Figure 3.

Solution: Let us denote

$$\begin{aligned} v_0 &= 3.0\,m/s \;\; \text{initial speed of the two rocks} \\ h &= 10\,m \;\; \text{height of the bridge} \\ g &= 9.8\,m/s^2 \;\; \text{gravity acceleration} \end{aligned}$$

We choose Cartesian coordinate system so that y-axis is vertical with positive direction up, and $y = 0$ corresponds to the water level (Fig. 3). We denote variables corresponding to Heather's and Jerry's rocks with subscripts H and J, respectively. The acceleration for both rocks is due to gravity only, and in our coordinates it is expressed as

$$\mathbf{a}(t) = -g\hat{\boldsymbol{\jmath}},$$

where $g = 9.8$ m/s^2. By integrating acceleration over time we find expressions for the velocity

$$\mathbf{v}(t) = -gt\hat{\boldsymbol{\jmath}} + \mathbf{v}_0,$$

and the position

$$\mathbf{y}(t) = -\frac{gt^2}{2}\hat{\boldsymbol{\jmath}} + \mathbf{v}_0 t + \mathbf{y}_0$$

for the two rocks, where $\mathbf{v}_0$ and $\mathbf{y}_0$ are constants of integration. These constants are determined from the initial conditions for each rock:

$$\mathbf{v}_H(0) = -3 \text{ m/s } \hat{\boldsymbol{\jmath}} \;\Rightarrow\; \mathbf{v}_{H0} = -3 \text{ m/s } \hat{\boldsymbol{\jmath}}$$
$$\mathbf{v}_J(0) = +3 \text{ m/s } \hat{\boldsymbol{\jmath}} \;\Rightarrow\; \mathbf{v}_{J0} = +3 \text{ m/s } \hat{\boldsymbol{\jmath}}$$
$$\mathbf{y}_H(0) = \mathbf{y}_J(0) = +10 \text{ m } \hat{\boldsymbol{\jmath}} \;\Rightarrow\; \mathbf{y}_{H0} = \mathbf{y}_{J0} = +10\text{m } \hat{\boldsymbol{\jmath}}$$

Since the entire motion takes place along the y-axis in what follows we drop the vector notation.

(a) Let t_H and t_J represent the times at which Heather's and Jerry's rocks, respectively, hit the water. Then,

$$y_H(t_H) = -\frac{gt_H^2}{2} + v_{H0}t_H + y_{H0} = 0 \quad \Rightarrow \quad -4.9t_H^2 - 3t_H + 10 = 0 \tag{3}$$

and

$$y_J(t_J) = -\frac{gt_J^2}{2} + v_{J0}t_J + y_{J0} = 0 \quad \Rightarrow \quad -4.9t_J^2 + 3t_J + 10 = 0 \tag{4}$$

By solving the two quadratic equations we obtain:

$$t_H = \frac{-3 + \sqrt{205}}{9.8} \text{ s} \quad \text{and} \quad t_J = \frac{3 + \sqrt{205}}{9.8} \text{ s}$$

The time elapsed between the two splashes is

$$\Delta t = t_J - t_H = \frac{6.0}{9.8} \text{ s} = 0.61 \text{ s}$$

(b) The velocity with which each stone hits the water is found when we substitute the time of impact into the expression for the velocity. We obtain

$$v_H(t_H) = -gt_H + v_{H0} = \left(-9.8\frac{-3 + \sqrt{205}}{9.8} - 3\right) \text{ m/s} = -\sqrt{205} \text{ m/s} = -14 \text{ m/s}$$

$$v_J(t_J) = -gt_J + v_{J0} = \left(-9.8\frac{3 + \sqrt{205}}{9.8} + 3\right) \text{ m/s} = -\sqrt{205} \text{ m/s} = -14 \text{ m/s}$$

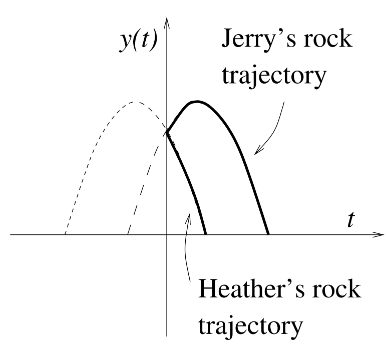

Figure 4.

(c) The velocities of the two rocks as they hit the water are the same. This can be understood from the symmetry of the two trajectories, (3) and (4). Since the motion takes place with constant acceleration both trajectories are parabolae. The trajectory of Jerry's rock (4) is the same as the trajectory of Heather's rock (3), only translated along the time axis for 0.61 seconds, as shown in Figure 4.

5. [25] Two identical stones are dropped from a tall building one after another. The first stone is dropped at time instant t_0, and the second one Δt later. Find an analytical expression for the verical distance between the two stones. Discuss your result. Will the separation between the stones increase or decrease? In your calculations ignore air resistance.

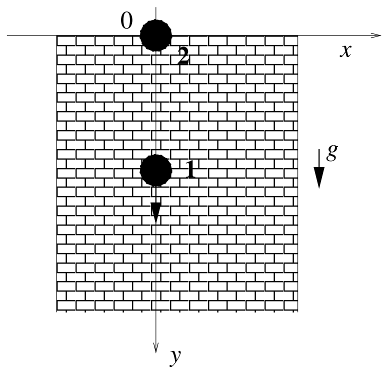

Figure 5.

Solution: Let a, v, and y represent acceleration, velocity, and vertical position of the two stones. The subscripts 1 and 2 refer to the first and second stone, respectively. We choose coordinate system so that y-axis is vertical, positive direction is down, and $y = 0$ at the top of the building. We also set $t_0 = 0$ without loss of generality. Since the two stones are dropped, not thrown, their initial velocities are zero.

In a similar way like in the previous problem we find kinematic expressions for the first stone

$$\begin{aligned} \mathbf{a}_1(t) &= g\hat{\boldsymbol{\jmath}} \\ \mathbf{v}_1(t) &= gt\hat{\boldsymbol{\jmath}} \\ \mathbf{y}_1(t) &= \frac{g}{2}t^2\hat{\boldsymbol{\jmath}} \end{aligned}$$

For the second stone $\mathbf{y}_2(t) = \mathbf{0}$ for $t \leq \Delta t$, and

$$\begin{aligned} \mathbf{a}_2(t) &= g\hat{\boldsymbol{\jmath}} \\ \mathbf{v}_2(t) &= g(t - \Delta t)\hat{\boldsymbol{\jmath}} \\ \mathbf{y}_2(t) &= \frac{g}{2}(t - \Delta t)^2\hat{\boldsymbol{\jmath}} \end{aligned}$$

for $t > \Delta t$. The separation between the two stones is

$$\Delta\mathbf{y}(t) = \mathbf{y}_1 - \mathbf{y}_2 = \begin{cases} \dfrac{gt^2}{2} & \text{for } t \leq \Delta t \\ (g\Delta t)\,t - \dfrac{g\Delta t^2}{2} & \text{for } t > \Delta t \end{cases}$$

Before the second stone was dropped ($t < \Delta t$), the separation grows quadratically. After the second stone was dropped ($t > \Delta t$) the separation keeps increasing, but now as a linear finction in time. From the moment it is dropped the first stone will always have higher speed than the second one.

End of examination
Total pages: 9
Total marks: 125