Given name:____________________ **Family name:**____________________

GTID:____________________ **Signature:**__________________________

Georgia Institute of Technology
School of Physics

Test Form 202L

PHYS 2211 (Intro to Physics)
Instructor: Slaven Peleš

Duration: 80 minutes

Standard calculators allowed

This examination paper consists of **8** pages and **5** questions. Please bring any discrepancy to the attention of a proctor. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

1. Print your name, Georgia Tech ID number, and sign the test form.
2. In each problem first define all quantities (e.g. v_{01} is initial velocity of the first object, a is the acceleration, ...) before proceeding with your calculations.
3. Show all the steps of your calculation and provide explanations when necessary. If you need more space, continue working on the back of the test form sheet.
4. Explain the physical meaning of your results.

Scores will be posted on your class website. Quiz grades become final when the next test is given.

1. [25] A 55 kg physicist stands on a scale in an elevator in outer space. She does not know whether the elevator is acclerating in free space or hanging in a gravitational field, but the scale reads 236 N. How long will an apple take to reach the floor if she drops it from rest (relative to herself), 1.9 m from the floor?

   Solution: Let us denote the apparent weight of the physicist with w, and her mass with m. The apparent gravity acceleration in the elevator frame is then obtained as

   $$a = \frac{w}{m} = \frac{236N}{55kg} = 4.29m/s^2$$

   The acceleration is in the vertical direction pointing downward.

   If we choose coordinate system such that y-axis is vertical, with positive direction up, and $y = 0$ to be at the floor of the elevator, we find the expression for the position of the apple to be

   $$y(t) = -\frac{1}{2}at^2 + h,$$

   where $h = 1.9$ m is the initial position of the apple in the elevator frame, before it was dropped. At time t_f, when the apple hits the floor of the elevator

   $$y(t_f) = 0 \;\Rightarrow\; -\frac{1}{2}at_f^2 + h = 0$$

   and therefore

   $$t_f = \sqrt{\frac{2h}{a}} = \sqrt{\frac{2 \times 1.9\,\mathrm{m}}{4.29\,m/\,s^2}} = 0.94\,\mathrm{s}$$

   The apparent gravity acceleration in the elevator frame is smaller than the gravity acceleration on the Earth, so the apple takes longer time to fall to the elevator floor.

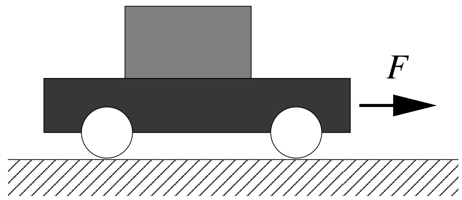

Figure 1.

2. [25] A suitcase of mass m_1 lies horizontally on a cart of mass m_2 (Figure 1). The coefficient of static friction between the suitcase and the cart is μ_s, and the coefficient of rolling friction between cart's wheels and the ground is μ_r. Express your results in terms of m_1, m_2, μ_s, μ_r and gravitational acceleration g.

   (a) Find an expression for the maximum acceleration of the cart for which the suitcase does not slip.

   (b) Find an expression for the force that would produce that acceleration.

   (c) Draw free body diagrams for the cart and for the suitcase.

   (d) Discuss your results.

Solution:

(a) If the suitcase does not slip, it means its acceleration is the same as the acceleration of the cart. Since the only force that pulls the suitcase horizontally is the static friction force, the maximum acceleration is determined by the static friction threshold

$$a_{\max} = \frac{f_{\max}}{m_1} = \frac{\mu_s m_1 g}{m_1} = \mu_s g.$$

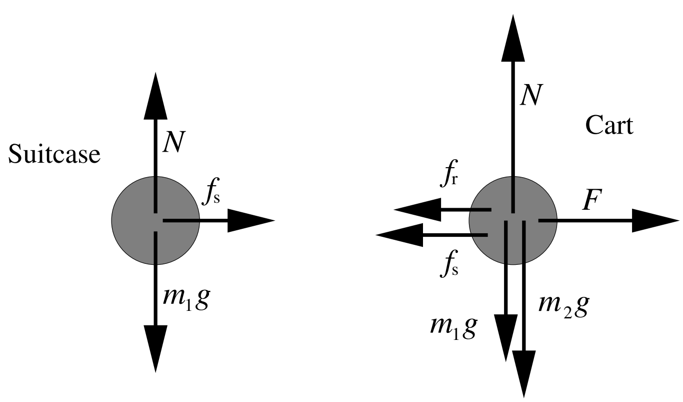

Figure 2.

(b) If the cart moves with acceleration $a_{\max}$ the cart and the suitcase move together, and can be considered as a single system. The forces acting on the cart/suitcase system are the force F, which pulls the cart, and the rolling friction force between cart wheels and the ground. From Newton's Second Law we obtain the equation of motion for the system

$$(m_1 + m_2)a_{max} = F - \mu_r(m_1 + m_2)g$$

and from there we find the expression for the force when $a = a_{\max}$ to be

$$F = (\mu_r + \mu_s)(m_1 + m_2)g$$

Here we chose x-axis to be horizontal, and positive direction to be in the direction of the force F.

(c) See Figure 2.

(d) The only horizontal force acting on the suitcase is a friction force. When the cart accelerates, it is the static friction that will try to pull the suitcase together with the cart. When the cart's acceleration is larger than $a_{\max}$, the static friction threshold is exceeded, and the suitcase will start sliding backward relative to the cart. The suitcase will be dragged by the cart for a while, due to kinetic friction force.

3. [25] Two balls of equal size are falling downwards at their terminal velocities. One ball is made of wood and has mass 1 kg, while the other is made of lead and has mass 10 kg. At time t_0 both balls are found to be 30 m above the ground. The radius of each ball is 6.0 cm.

   (a) Find how much time will pass from t_0 before each ball hits the ground?

   (b) Discuss your results. Which ball hits the ground first? Why?

   Solution:

   (a) When an object moves at its terminal velocity that means that the drag force balances all the other forces acting upon the object. For a ball freely falling through the air that can be expressed as

$$\frac{1}{4} A\, v_{term}^2 = mg$$

   where v_{term} is the terminal speed, A is the area of the ball's cross section, m is the mass of the ball, and g is acceleration due to gravity. From this expression we find the expression for the terminal speed as

$$v_{term} = \sqrt{\frac{4mg}{A}}.$$

   For the lead ball the terminal velocity is

$$v_{lt} = \sqrt{\frac{4 \times 10 \times 9.8}{\pi (0.06)^2}}\,\mathrm{m/s} = 186\,\mathrm{m/s},$$

   and for the wooden ball it is

$$v_{wt} = \sqrt{\frac{4 \times 1 \times 9.8}{\pi (0.06)^2}}\,\mathrm{m/s} = 58.9\,\mathrm{m/s}.$$

   Since the total force acting on each ball is zero, the balls move with constant velocities and the time that takes them to hit the ground is

$$t_l = h/v_{lt} = 0.16s \quad \text{and} \quad v_{wt} = h/v_{lt} = 0.51s,$$

   for the lead and wooden ball, respectively. Here $h = 30$ m is the hight at which the two balls are found initially.

   (b) The lead ball has larger terminal velocity, and therefore hits the ground first. Heavier ball needs larger drag force to stop its acceleration. Since the two balls have the same size, the heavier ball has to move at higher speed in order to produce drag force which will be equal to its weight.

4. [25] Compressed air is used to fire a 50 g ball vertically upward from a 96 cm tall tube. Initially the ball is at rest at the bottom of the tube. The air exerts an upward force of 2.0 N on the ball as long as it is in the tube. How high does the ball go above the top of the tube? Assume acceleration due to gravity is 9.8 m/s^2. Explain your result.

Solution: Inside the tube equation of motion for the ball is given by:

$$m\mathbf{a} = \mathbf{F}_{air} + m\mathbf{g},$$

where m is mass of the ball, $\mathbf{a}$ is acceleration of the ball inside the tube, $\mathbf{F}_{air}$ is the force that air exerts on the ball, and $\mathbf{g}$ is acceleration due to gravity. If we choose y-axis to be vertical and pointing up, we can find the acceleration of the ball inside the tube as

$$\mathbf{a} = \frac{\mathbf{F}_{air}}{m} + \mathbf{g} = \left(\frac{F_{air}}{m} - g\right)\hat{\boldsymbol{\jmath}} = \left(\frac{2.0\,\mathrm{N}}{0.050\,\mathrm{kg}} - 9.8\,\mathrm{m/s}^2\right)\hat{\boldsymbol{\jmath}} = 30.2\,\mathrm{m/s}^2\hat{\boldsymbol{\jmath}}.$$

Next, let us find velocity of the ball at the top of the tube. If we choose the origin of our coordinate system at the top of the tube, expressions for the position and the velocity of the ball become $y(t) = \frac{1}{2}at^2 - L$ and $v(t) = at$, respectively, where L is the length of the tube (here we dropped vector notation since the entire motion takes place along the y-axis). By eliminating time from these two equations, we find that the velocity of the ball at the top of the tube is

$$v_{top} = +\sqrt{\frac{2L}{g}} = +7.61\mathrm{m/s}.$$

Outside of the tube the only force acting on the ball is gravity, and the ball moves with constant acceleration $\mathbf{g}$. In our coordinate system the expression for the ball's position and velocity are

$$y(t) = -\frac{1}{2}gt^2 + v_{top}t \quad \text{and} \quad v(t) = -gt + v_{top},$$

respectively. When the ball reaches the highest point, its velocity is zero, so from the second equation we find that the time it takes the ball to travel from the top of the tube to its highest point is $t_{max} = v_{top}/g$. By substituting this time into the expression for the position $y(t)$, we find that

$$y_{max} = y(t_{max}) = \frac{v_{top}^2}{2g} = 2.96\mathrm{m}$$

The ball will go 2.96 m high above the top of the tube, what is approximately three times the length of the tube.

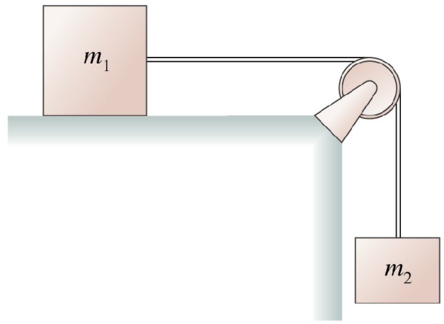

Figure 3.

5. [25] Two blocks of mass $m_1 = 2.5$ kg and $m_2 = 1.5$ kg are connected by a rope as shown in Figure 3. The system is initially at rest. The coefficients of static and kinetic friction between block m_1 and the surface on which it slides are $\mu_s = 0.1$ and $\mu_k = 0.02$ respectively. Take the acceleration due to gravity to be 9.8 m/s^2. Assume the rope and the pulley are massless, the rope does not stretch, and the pulley is frictionless.

   (a) Find the acceleration of the system.

   (b) Find the tension in the rope connecting the two blocks.

   (c) Discuss your results. Does the system move?

   Solution: (a) First, let us assume that the system does not move. In that case the static friction force acting on block m_1 has to be equal to gravity force acting on block m_2. That is possible only if

$$f_s = \mu_s m_1 g \geq m_2 g,$$

   (where g is the gravity acceleration). By substituting values for m_1, m_2, μ_s and g we find this is NOT the case:

$$2.45\,\text{N} \not\geq 14.7\,\text{N}.$$

   Therefore, the system will exhibit an accelerated motion.

   The two blocks will have the same acceleration a, since they are connected by the rope which does not stretch. From the Newton's Second Law we find the equations of motion for the two blocks to be

$$m_2 a = -T + m_2 g \tag{1}$$
$$m_1 a = T - \mu_k m_1 g \tag{2}$$

   where $T > 0$ is the tension force in the rope. By adding equations (1) and (2) we obtain:

$$a = \frac{m_2 - \mu_k m_1}{m_1 + m_2} g = 3.55\,\text{m/s}^2$$

   (b) By substracting equation (1) from the equation (2) we obtain the expression for the tension in the rope

$$T = \frac{(\mu_k + 1) m_1 m_2}{m_1 + m_2} g = 9.37\,\text{N}$$

(c) Since the rope does not stretch the magnitude of the tension force is constant along the entire length of the rope. The direction of the tension force changes accross the pulley. The system moves because the static friction threshold is smaller than the tension in the rope.

End of examination
Total pages: 8
Total marks: 125