**Given name:**____________________ **Family name:**____________________

**GTID:**____________________ **Signature:**__________________________

**Georgia Institute of Technology**
**School of Physics**

**Test Form 303L**

**PHYS 2211 (Intro to Physics)**
Instructor: Slaven Peleš

**Duration: 80 minutes**

**Standard calculators allowed**

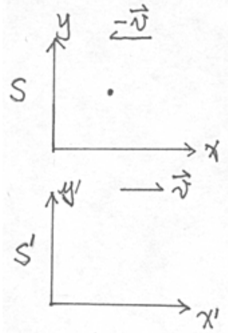
This test form consists of **6** pages and **5** questions. Please bring any discrepancy to the attention of a proctor. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

1. Print your name, Georgia Tech ID number, and sign the test form.
2. In each problem first define all quantities (e.g. $v_{01}$ is initial velocity of the first object, $a$ is the acceleration, ...) before proceeding with your calculations.
3. Show all the steps of your calculation and provide explanations when necessary. If you need more space continue working on the back of the test form sheet.
4. Explain the physical meaning of your results.

Scores will be posted on your class website. Quiz grades become final when the next test is given.

1. [25] A firecracker explodes in reference frame $S$ at $t = 1.00$ s. A second firecracker explodes at the same position at $t = 5.00$ s. In reference frame $S'$, which moves in the $x$-direction at speed $v$, the first explosion is detected at $x' = 3.00$ m and the second at $x' = -3.00$ m. What is the velocity of frame $S'$ relative to frame $S$? Explain.




| | S frame | S' frame |
|---|---|---|
| 1st explosion | $(x_1, y_1, t_1)$ | $(x_1', y_1', t_1')$ |
| 2nd explosion | $(x_2, y_2, t_2)$ | $(x_2', y_2', t_2')$ |

When S' frame moves at $v$ in $x$-direction respective to S frame, S frame is moving at $v$ but in the opposite direction relative to S' frame. Hence, the following relations:

$$\begin{cases} x_1' = x_1 - vt \\ x_2' = x_2 - vt \end{cases}$$

$$\Rightarrow (x_1' - x_2') = (x_1 - x_2) + v(t_2 - t_1)$$

$\because$ $x_1' = 3.00$m, $x_2' = -3.00$m, $x_1 = x_2$ $t_1 = 1.00$s, $t_2 = 5.00$s

$$\Rightarrow v = \frac{(x_1' - x_2') - (x_1 - x_2)}{(t_2 - t_1)} \doteq \frac{6}{4} = 1.5\,\text{m/s}$$

$\therefore$ The magnitude of the velocity is 1.5 m/s, and the direction is in the +X direction of S frame.

2. [25] Assuming that Jupiter's orbit around the Sun is a circle, find the length of Jupiter's year in terms of Earth's years. The mass of the Sun is $2.0 \times 10^{30}$ kg, mass of Jupiter is $1.9 \times 10^{27}$ kg, and the average distance between Jupiter and the Sun is $7.8 \times 10^{8}$ km. Take the universal gravitational constant to be $6.67 \times 10^{-11}$ $\text{Nm}^2/\text{kg}^2$.

$G \to$ universal gravity constant

$M \to$ mass of the Sun

$m \to$ mass of the Jupiter

$r \to$ average distance between Jupiter and the sun

$\omega \to$ angular speed of Jupiter around the sun

Universal Force:

$$F = G \cdot \frac{M \cdot m}{r^2}$$

Centrifugal Force:

$$F = m \cdot \omega^2 \cdot r$$

$$\Rightarrow G \frac{M \cdot m}{r^2} = m \cdot \omega^2 r$$

$$\Rightarrow \omega = \left(\frac{G \cdot M}{r}\right)^{1/2}$$

Hence, Jupiter's year

$$T = \frac{2\pi}{\omega} = 2\pi \cdot \left(\frac{r}{G \cdot M}\right)^{1/2}$$

$$= 2 \cdot 3.14 \left(\frac{7.8 \times 10^{11}\,\text{m}}{6.67 \times 10^{-11}\,\text{Nm}^2/\text{kg}^2 \cdot 2 \times 10^{30}\,\text{kg}}\right)^{1/2}$$

$$= 3.75 \times 10^{8}\,\text{s}$$

$$= 3.75 \times 10^{8}\,\text{s} / [(365 \cdot 24 \cdot 60 \cdot 60)\,\text{s/year}]$$

$$= 11.88 \text{ years}$$

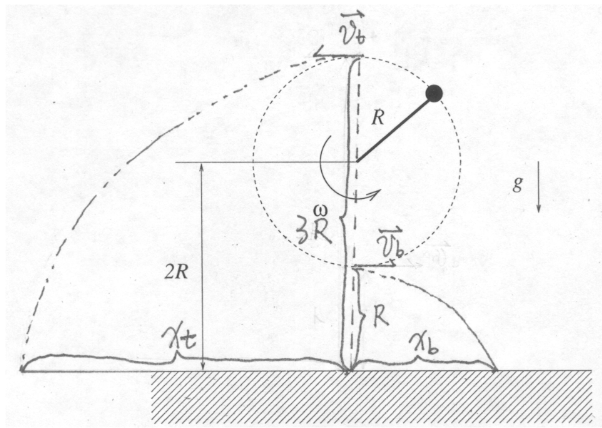



**Figure 1**.

3. [25] A steel ball rotates in a vertical plane while attached to one end of a rigid rod of length $R$, as shown in Figure 1. The other end of the rod is attached to a rotating pivot located at a distance $2R$ above the ground. The angular velocity of the system is constant. At some point in time the ball is released from the rod and falls to the ground. Find the ratio between the horizontal distances traveled by the ball (before hitting the ground) when it is released from the top of its circular path and when it is released from the bottom of its circular path.

| | Ball from top | Ball from bottom |
|---|---|---|
| Herizontal velocity | $v_t=\omega R$ | $v_b=\omega R$ |
| Vertical distance | $h_t=3R$ | $h_b=R$ |
| Travelling time | $t_t=?$ | $t_b=?$ |
| Horizontal distance | $x_t=?$ | $x_b=?$ |

$h_t=\frac{1}{2}\cdot g\cdot t_t^2$ $\qquad$ $h_b=\frac{1}{2}\cdot g\cdot t_b^2$

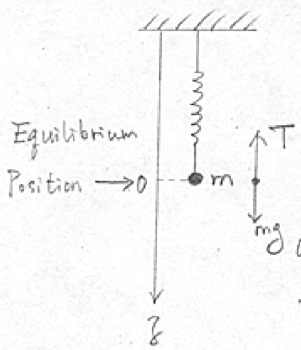
$x_t=v_t\cdot t_t$ $\qquad$ $x_b=v_b\cdot t_b$

$$\therefore \frac{x_t}{x_b}=\frac{v_t\cdot t_t}{v_b\cdot t_b}=\frac{v_t}{v_b}\left(\frac{2\cdot h_t/g}{2\cdot h_b/g}\right)^{1/2}=\frac{\omega R}{\omega R}\cdot\left(\frac{3R}{R}\right)^{1/2}=\sqrt{3}$$

4. [25] A 2.1 kg mass is hung vertically on an elastic spring with elasticity constant of 110 N/m. The mass is pulled down 6 cm from its equilibrium position and then released from rest.

   (a) Find the direction and the magnitude of the total force that is acting upon the mass at the time instant 4 seconds later.

   (b) Find the direction and the magnitude of the velocity at that instant.




For oscillation object, displacement $z$ satisfies:

$$z = A\cos\omega t$$

where $A$ is amplitude, $\omega$ is angular frequency, and $\omega = \sqrt{\frac{k}{m}}$, $k$ is the elastic constant of the spring, $m$ is the mass of the object.

Since the mass is released from rest 6 cm from its equilibrium position, then $A = 0.06$ m.

(a) The total force acting on the mass $F = mg - T$,

and $F = ma = m\frac{d^2 z}{dt^2} = -A\omega^2\cos\omega t$

So at $t = 4s$

$$F = -0.06m \cdot \left(\frac{110N/m}{2.1kg}\right) \cdot \cos\left[\left(\frac{110N/m}{2.1kg}\right)^{1/2} \cdot 4s\right] = 5.10\ N$$

Since $F > 0$, $\therefore$ the direction is downwards.

(b) $v = \frac{dz}{dt} = -A\omega\sin\omega t$

$\therefore$ at $t = 4s$, $v = -(0.06m) \cdot \left(\frac{110N/m}{2.1\ kg}\right)^{1/2} \cdot \sin\left[\left(\frac{110N/m}{2.1kg}\right)^{1/2} \cdot 4s\right]$

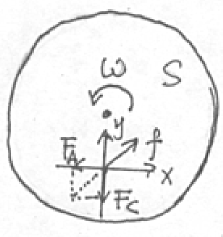
$$v = -(0.06)\left(\frac{110N/m}{2.1kg}\right) \cdot \sin\left[\left(\frac{110N/m}{2.1\ kg}\right)^{1/2} \cdot 4s\right] = 0.28\ m/s$$

the direction is also downwards.

5. [25] A coin of mass 15 g is lying on a horizontal turntable, 12 cm away from its center. The coin is lubricated so the coefficient of static friction between the table and the coin is only 0.02. The table starts rotating, accelerationg from rest at the rate of 0.5 rad/s$^2$.

   (a) How long it will take before the coin starts slipping, measured from the instant when the table starts moving?

   (b) How many revolutions will the table make before the coin starts slipping (Note that the result may not be an integer number).




In rotating frame S, forces acting on the mass are following:

Friction: $f = \mu m g$

Contrifugual Force: $F_c = m\omega^2 r$

Azimuthal Force: $F_A = m\dot{\omega} r$ where $\omega = \int_0^t \dot{\omega}\,dt = \dot{\omega} t$

Before the mass begins to slip, the above force satisfy:

$$\vec{F_A} + \vec{F_c} = -\vec{f}$$

$$\Rightarrow |F_A|^2 + |F_c|^2 = |f|^2 \quad \text{(Because } \vec{F_A} \text{ \& } \vec{F_c} \text{ are orthogonal)}$$

$$\Rightarrow m^2\omega^4 r^2 + m^2\dot{\omega}^2 r^2 = \mu^2 m^2 g^2$$

$$\Rightarrow \omega^4 r^2 + \dot{\omega}^2 r^2 = \mu^2 g^2$$

$$\Rightarrow \dot{\omega}^4 t^4 r^2 + \dot{\omega}^2 r^2 = \mu^2 g^2$$

$$\Rightarrow t = \left(\frac{\mu^2 g^2 - \dot{\omega}^2 r^2}{\dot{\omega}^4 \cdot r^2}\right)^{1/4}$$

(a) At the instant when the mass is about to slip, $\mu = 0.02$, hence

$$t = \left(\frac{(0.02)^2 \cdot (9.8)^2 - (0.5)^2 \cdot (0.12)^2}{(0.5)^4 \cdot (0.12)^2}\right)^{1/4} = 2.49\,s$$

(b) $\theta = \int_0^t \omega \cdot dt = \int_0^t \dot{\omega} t\,dt = \frac{1}{2}\dot{\omega} t^2 = \frac{1}{2}(0.5)\cdot(2.49)^2 = 1.55$ rad

Revolutions: $\frac{\theta}{2\pi} = \frac{1.55}{2 \cdot 3.14} = 0.25$



**End of examination**
**Total pages: 6**
**Total marks: 125**