Given name:____________________ **Family name:**____________________

GTID:____________________ **Signature:**__________________________

Georgia Institute of Technology
School of Physics

Solutions to Quiz 505L (prepared by Yang Ding)

PHYS 2211 (Intro to Physics)
Instructor: Slaven Peleš

Duration: 80 minutes

Standard calculators allowed

This test form consists of **9** pages and **5** questions. Please bring any discrepancy to the attention of a proctor. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

1. Print your name, Georgia Tech ID number, and sign the test form.
2. In each problem first define all quantities (e.g. v_{01} is initial velocity of the first object, a is the acceleration, ...) before proceeding with your calculations.
3. Show all the steps of your calculation and provide explanations when necessary. If you need more space continue working on the back of the test form sheet.
4. Explain the physical meaning of your results.

Scores will be posted on your class website. Quiz grades become final when the next test is given.

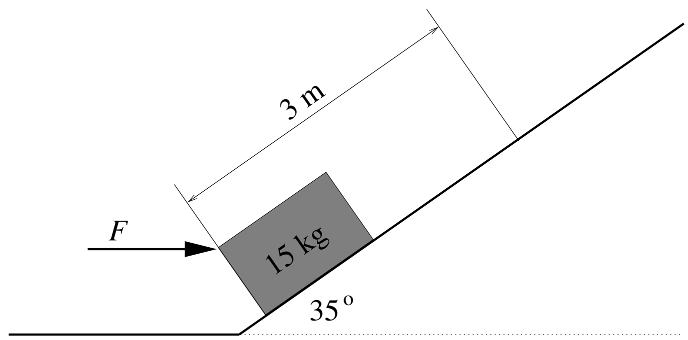

Figure 1.

1. [25] A 15 kg block is pushed 3.0 meters up a frictionless plane by a horizontal force of magnitude$F = 250$ N, as shown in Figure 1. The plane is inclined for 35 degrees above the horizontal. Take the acceleration due to gravity to be 9.8 m/s^2

   (a) What is the work that force F does on the block?

   (b) What is the work that gravity does on the block?

   (c) Calculate the work that normal force from the plane does on the block.

   (d) Discuss your results. Which forces do positive and which negative work?

Solution

The directions of the force and the displacement are shown in the Figure 2. Let W_F, W_g, W_n represent the work done by the force F, gravity and the normal force from the plane.

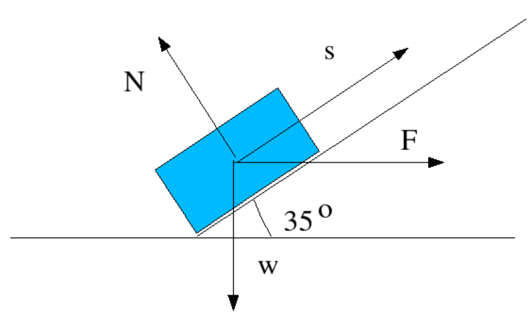

Figure 2.

Since all forces are constant the work can be calculates as

$$W_{12} = \int_1^2 \overrightarrow{F} \cdot d\overrightarrow{s} = \overrightarrow{F} \cdot \int_1^2 d\overrightarrow{s} = \overrightarrow{F} \cdot \overrightarrow{s_{12}} \tag{1}$$

(a) The work done by the F is:

$$W_f = \vec{F} \cdot \vec{s} = 250N \times 3.0m \times \cos 35^\circ = 614J$$

(b) The work done by the gravity is:

$$\begin{aligned} W_g &= \vec{w} \cdot \vec{s} \\ &= 9.8m/s^2 \times 1.5kg \times 3m \times \cos(90^\circ + 35^\circ) \\ &= -252J \end{aligned}$$

(c) The work done by the normal force is:

$$W_n = \vec{N} \cdot \vec{s} = 0$$

(d) The work done by the gravity is *negative* and with a magnitude the same as the increase of the gravitational potential, which is $mgs \times sin 35^\circ$. Because the normal force is also normal to the displacement, the work done by the normal force is zero. And the work done by the force F is *positive*.

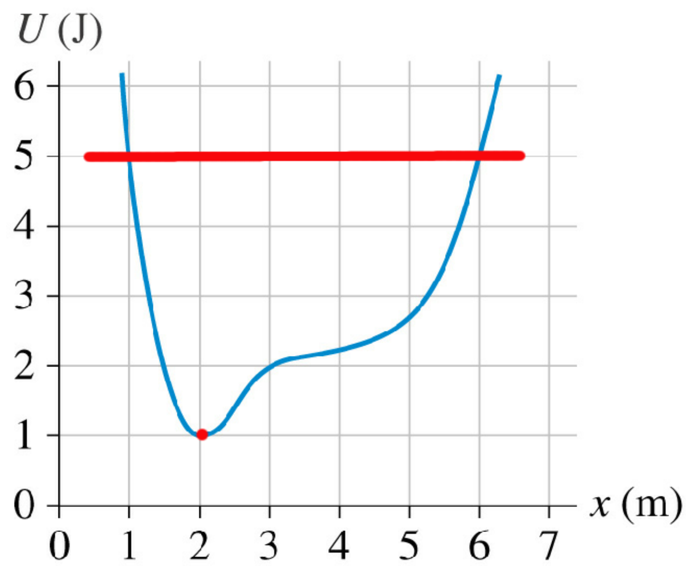

Figure 3.

2. [25] Figure 3 shows the potential energy diagram for a 0.2 kg particle released from rest at position $x = 1$ m.

   (a) What are the turning points of the motion? Explain.

   (b) At what position will the particle have its maximum speed? Explain.

   (c) What is the particle's maximum speed?

Solution:

Let m represent the mass of the particle and v represent the its speed.

(a) At x=1m, the potential of the particle is $U(x = 1) = 5$J and its total energy is also $E = 5$J since it is released from rest. After that, the particle will move in positive x-direction because the potential decrease in positive x-direction. At the turning point, the velocity of the particle must be zero. Due to energy conservation, the potential at the turning points must be 5J. So the turning points are $x = 1$ m and $x = 6$ m. The particle will go forward and back between $x = 1$ m and $x = 6$ m. Note that since the potential is not necessarily the gravitational potential, the x may be other than height.

(b) The total energy of the particle is conserved. So when its potential So when $U(x)$ reach its minimum, the speed will reach its maximum. The figure indicate that the minimum of $U(x)$ is 1J when $x = 2$ m.

(c) Let v_{max} represent the maximum speed. From energy conservation it follows:

$$E = U(x = 2) + \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{2[E - U(x = 2)]}{m}} = 6.32\,\text{m/s}$$

So the maximum speed of the particle is 6.32 m/s.

3. [25] A crane has an engine connected to a pulley-cable system that hoists a bucket of cement with a total weight of 20,000 N to a height of 40 m. If there is a frictional retarding force of 5000 N acting in the pulley-cable, what is the total power supplied by the engine if it pulls the load upward at constant speed of 3.0 m/s?

Solution:

There are three forces exert on the bucket: the gravity, the retarding force and the force from the system. We use $\vec{W}$, $\vec{F_r}$ and $\vec{F_s}$ to represent them. Because the bucket is moving at a constant speed, the net force acting on it is zero, that is:

$$\vec{W} + \vec{F_r} + \vec{F_s} = 0.$$

Since they are all vertical, take upward direction as positive direction and we have:

$$\begin{aligned} -|\vec{W}| - |\vec{F_r}| + |\vec{F_s}| &= 0, \\ |\vec{F_s}| &= |\vec{W}| + |\vec{F_r}|, \\ |\vec{F_s}| &= 5000N + 20,000N = 25,000N. \end{aligned}$$

Let P represent the total power of the system, then

$$P = \vec{F_s} \cdot \vec{v}, \text{ where } \vec{v} \text{ is the velocity of the bucket.}$$

The velocity of the bucket and the force from the system are in the same direction, so the total power of the system is just:

$$P = |\vec{F_s}||\vec{v}| = 25,000N \times 3.0m/s = 75,000N \cdot m/s$$

4. [25] A spacecraft of mass m is orbiting a planet of mass M in a circular orbit of radius R. What is the minimum additional energy required to send the spacecraft far away from the planet? Explain.

Solution:

Let v represent the speed of the spacecraft and a represent the magnitude of the acceleration of it. And let w be the magnitude of the gravitational force on the spacecraft. When the spacecraft orbiting in a circle, the acceleration is:

$$a = \frac{v^2}{R}$$

And the only force acting on the spacecraft is the gravity. So from Newton's second law we have:

$$w = ma.$$

And together with the Newton's law of gravity

$$w = \frac{GMm}{R^2}\text{(G is the gravitational constant)}$$

we obtain:

$$w = m\frac{v^2}{R} = \frac{GMm}{R^2}$$
$$\Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

We can see that on the left hand of the last equation is just the kinetic energy of the spacecraft.

Let U_g represent the gravitational potential at R, and

$$U_g = -\frac{GMm}{R}.\text{(We have chosen the potential be zero at } r \to \infty)$$

To send the spacecraft far away form the planet means we are going to put the spacecraft at a point where gravitational potential is nearly zero. There the total mechanical energy of the spacecraft is at least zero because it may also have non-negative kinetic energy. And when the spacecraft is on a orbit of radius of R, the total mechanical energy is only:

$$\frac{1}{2}mv^2 + U_g = \frac{GMm}{2R} + (-\frac{GMm}{R}) = -\frac{GMm}{2R}$$

So we have to add energy at least of

$$0 - (-\frac{GMm}{2R}) = \frac{GMm}{2R}$$

to send the spacecraft far away.

Discussion:
When choose the zero point of the gravitational potential at infinite far, the gravitational potential is negative. And the total mechanical energy of the spacecraft is also negative if the spacecraft is orbiting the planet with circular orbit. So to send the spacecraft far away from the planet, we have to add extra energy to it.

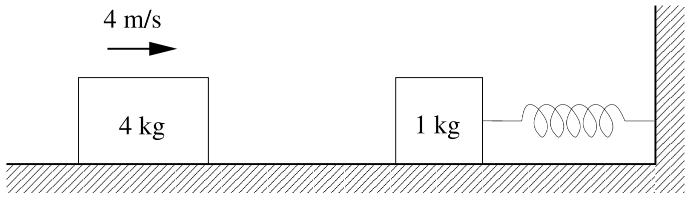

Figure 4.

5. [25] A spring-mass system consisting of a 1 kg block and a spring with elasticity constant 300 N/m is initially in its equilibrium position (Figure 4). The 1 kg block undergoes a collision with a 4 kg block, which is moving at speed 4.0 m/s. After the collision the two blocks stick together. Assume that friction is negligible.

   (a) What is the maximum compression of the spring (in terms of length) after the collision?

   (b) What is the amount of mechanical energy lost in the collision?

   Solution:
   Use index of 1 to represent the 4kg block and index of 2 to represent the 1kg block. The mass of the two blocks are $m_1 = 4kg$ and $m_2 = 1kg$.
   Let v_{b1} and v_{b2} represent the velocities before collision: $v_{b1} = 4m/s$, $v_{b2} = 0m/s$.
   And let v_{a1} and v_{a2} represent the velocities right after collision.

   (a) Consider the two blocks as a system, then the only force in horizontal direction exert on the system during the collision is from the spring. But since the collision happens very quickly, the impulse form the spring is negligible. So the momentum of the system is reserved:

$$m_1 v_{b1} + m_2 v_{b2} = m_1 v_{a1} + m_2 v_{a2}.$$

   After the collision the two blocks stick together, so they have the same velocity:

$$v_{a1} = v_{a2}.$$

   From these two equations and plug in the values we obtain:

$$\begin{gathered} 4kg \times 4m/s + 1kg \times 0m/s = 4kg \times v_{a1} + 1kg \times v_{a1}, \\ \Rightarrow v_{a1} = 3.2m/s \end{gathered}$$

   And then the spring is compressed by the two blocks while the two blocks slow down. During the compression, the mechanical energy is conserved, that's the sum of the potential energy in the spring and the kinetic energy of the two blocks is conserved. So when the spring reaches its maximum compression (represented by l_{max}), the velocity of the two blocks reach the minimum 0m/s:

$$\frac{1}{2}(m_1+m_2)v_{a1}^2+\frac{1}{2}k(0m)^2=\frac{1}{2}(m_1+m_2)(0m/s)^2+\frac{1}{2}kl_{max}^2$$
$$\frac{1}{2}(4kg+1kg)(3.2m/s)^2=\frac{1}{2}\times 300N/m\times l_{max}^2$$
$$l_{max}=0.41m$$

(b) Let E_b represent the mechanical energy before the collision, and E_a represent the mechanical energy after collision.

$$E_b=\frac{1}{2}m_1v_{b1}^2+\frac{1}{2}m_2v_{b2}^2,$$
$$E_b=0.5\times 4kg\times(4m/s)^2+\frac{1}{2}\times 1kg\times(0m)^2,$$
$$E_b=32kg\cdot m^2/s^2,$$

$$E_a=\frac{1}{2}(m_1+m_2)v_{a1}^2,$$
$$E_a=0.5\times(4kg+1kg)\times(3.2m/s)^2$$
$$E_a=25.6kg\cdot m^2/s^2.$$

So the mechanical energy lost in the collision is:

$$E_b-E_a=32kg\cdot m^2/s^2-25.6kg\cdot m^2/s^2=6.4kg\cdot m^2/s^2$$

End of examination
Total pages: 9
Total marks: 125