MORE ON MICROCANONICAL PARADIGMS

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1. INTRODUCTION

Nonlinear physics presents us with a perplexing variety of complicated problem formulations and their strange solutions. Notable examples include Mandelbrot, one of our beards, swedish fish-eye soup breakfasts, far from equilibrium Belgians and even the lowly parabola. It is a fact that under the scrutiny of modern computation in the hands of world experts all of this problems can be made to show dirt, even on the smallest scales.

In general one deals with such problems by whatever means are at hand.

Perhaps the most perplexing facet of analytic description is the lack of smoothness. Consider for example the parabola[1].

$$f(x) = x^2 \qquad (1.1)$$

We have investigated this equation numerically by successive magnifications of the important regions of f(x). Careful numerical exploration, fig. 1.1, shows a dismaying lack of smoothness.

It is the point of this paper - actually a breakthrough - that a language of discourse exists in which a parabola really <u>is</u> smooth. As fig. 1.1 clearly indicates, (1.1) is not smooth. Now consider, for any f(x), the paradigmatic description

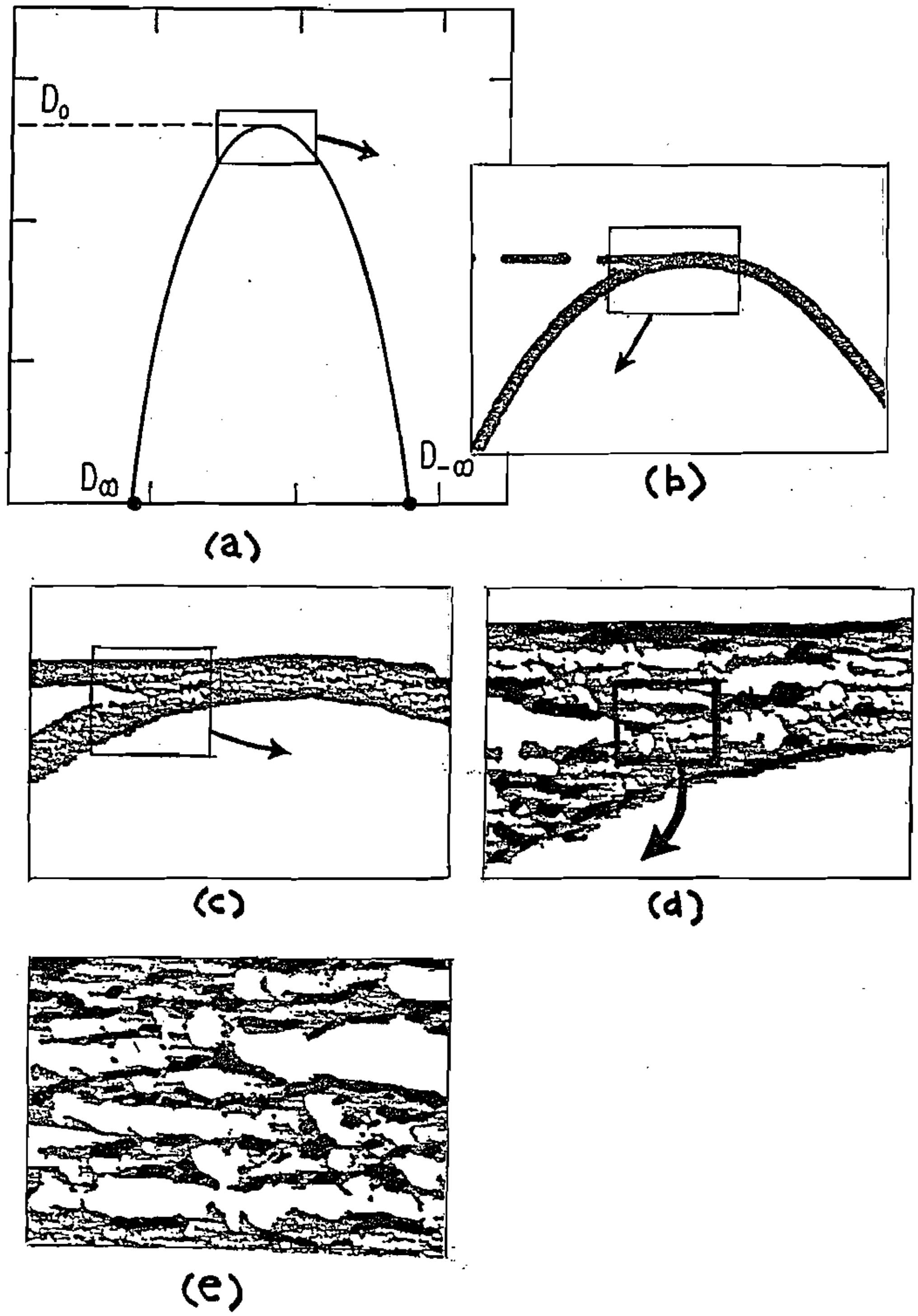

Fig. 1.1 A numerical exploration of the equation (1.1). For numerical expedience the parabola was approximated by the MP curve. The fit is excellent - the result is accurate to 1.4%, indicating a very fast convergence. We believe this is the most accurate calculation of parabola up to date and is certainly within the rigorous bounds of ref. [2].

$$f(x) \rightarrow \frac{\int dt\; f(f^{-1}(ff^{-1}\ldots\ldots\ldots ff^{-1}(f(t))\ldots\ldots)}{\int du\; g(g^{-1}\ldots\ldots g(g^{-1}(u)\ldots\ldots))}\; h(x) \qquad (1.2)$$

The function h is plotted in fig. 1.2. This plot _is_ _not_ just imagination. Rather, it represents 750 man hours of precise verification.

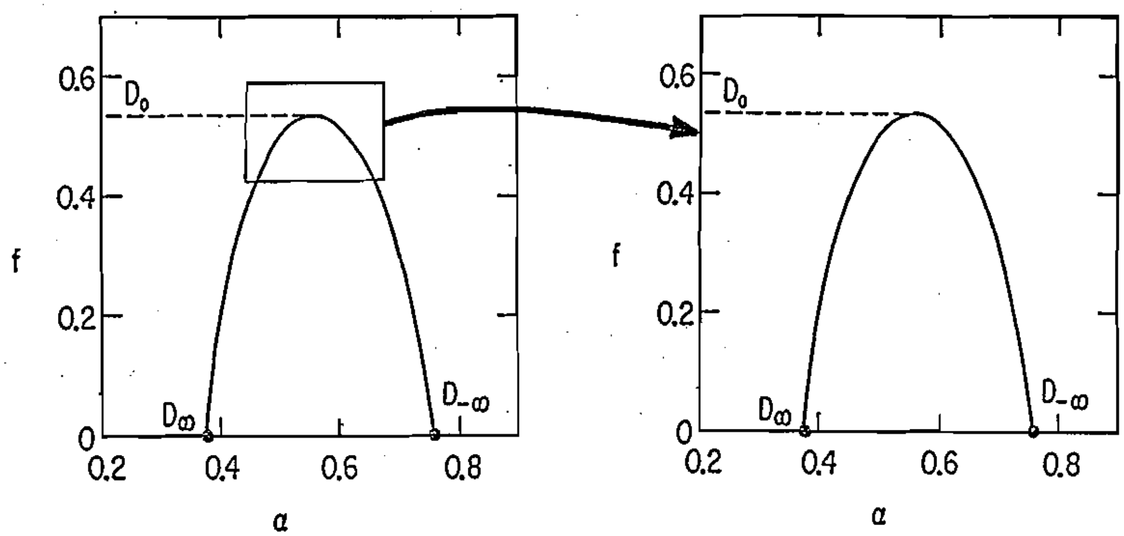

Fig. 1.2 The MP analysis of the parabola performed in quadruple precision on a VAX-750 minicomputer. Clearly a greater computational effort will be required to prove that this figure is trully correct.

However, numerical solutions of (1.2) are made difficult by subtle convergence problems, such as the persistent failure of 1 to converge to 0, even after thousands of iterations[3]. These convergence problems, as well as the difficulties facing us elsewhere, such as in the problem of understanding the fully developed turbulence, can be easily circumvented by the methods which will be introduced here.

An elegant formulation of these methods is afforded by the _microcanonical paradigm_.

By the law of large numbers, all combinatorics such as we shall encounter, is parabolic in the central region. Notice that above curve is precisely of this nature. This is a technical point.

The rest of our formalism attempts to unravel this complexity in a rancible fashion.

The microcanonical paradigm (hereafter referred to as MP) can be briefly explained as follows. There is a function f whose value is the answer. We can assume that f is either zero, or the function plotted in figs. 1.2 to 2.9. In this paper we assume that f is the answer. In the forthcoming 30 papers we will assume that f is zero.

The essence of MP can perhaps best be grasped by covering Anette Wad, the source of natural radiance, Fig. 1.3, by infimum of large balls. This turns Anette Wad into mashed potatoes, Fig. 1.4.

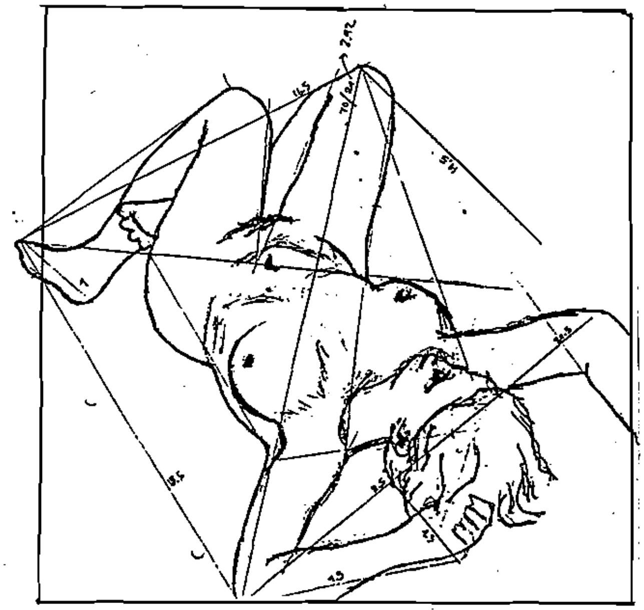

Fig. 1.3 Anette Wad before the application of the microcanonical paradigm.

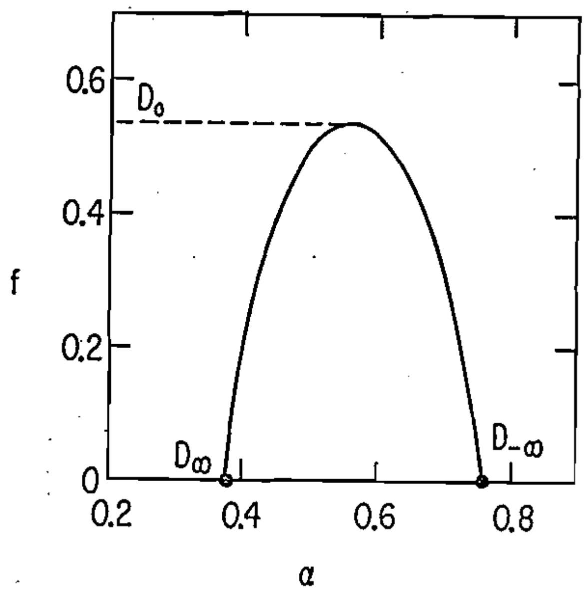

Fig. 1.4 Anette Wad after the application of the microcanonical paradigm.

Mashed potatoes (hereafter referred to as MP) are invariant under all coordinate transformations. They are very nice, because, unlike the complicated and incomprehensible scaling function of Feigenbaum[4], they are smooth and continuous. Not only that, but also their derivatives are smooth and continuous. The fact that also the derivatives of their derivatives are smooth and continuous makes them very smooth indeed. Not only that, but given any set of data, we obtain the same scaling function (cf. Figs. 1.2 to 2.6), thus attaining a remarkable degree of universality, in contradistinction to the everywhere discontinuous and therefore not nice and hard to use scaling functions such as those of ref. [5] which are different for every different problem. We shall hereafter refer to such nowhere nice scaling functions as SP (sour potatoes).

Examples of problems in which the MP universality ideas have had striking success are the parabola, the identity and zero. Maps of this type, familiarly known as class IIb, model a variety of physical systems: we refer the reader to papers 1 to 1007, hereafter referred to as I, II, III, IV, V, ..., MVII for detailed discussion of their physical applications. There is no known obstacle to deriving all of the known physics from the class IIb theories.

We give many examples of the procedures employed.

2 . THE NONTRIVIAL APPLICATIONS OF MP

The first example of a falfa curve, a round, nice looking and everywhere caressably smooth curve (even though the previously defined[6],[7] D(q) has still not converged for q = - 40), is the MP analysis of the group photograph of the authors, Fig. 2.1.

Our next example is afforded by the binning of 37 bordeaux performed by five connoisseurs[8] of french wines, Fig. 2.2. We bin the data by pouring the wine into sucessively smaller glasses (this is the principle of microbinning), covering the whole spectrum of scales from the size of universe to the quark size. French drinkers can be observed over this entire range of scales. The probability distribution is computed from the successive close passes of wine taster to the wine. This is known as the natural, or Ruelle-Bowen measure.

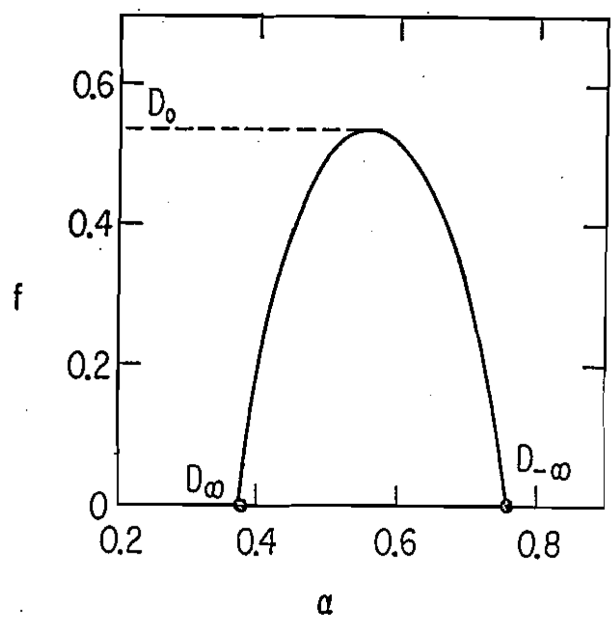

Fig. 2.1 The f minus α plot of the strenghts of the singularities for the DLA (dodderingly lethargic authors) clustering phenomenon.

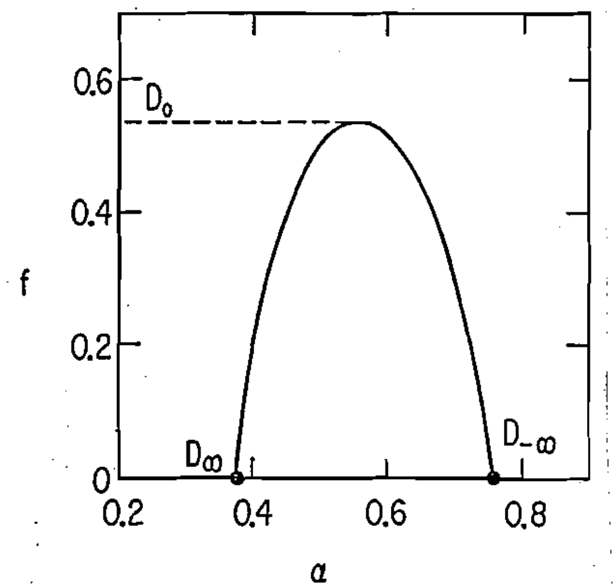

Fig. 2.2 A plot of the mashed potatoes function f versus α for bordeaux wines.

In Fig. 2.3 and Fig. 2.4 we plot the f - α curve for two fractal sets of wildly different ramifications. Note that the microcanonical paradigm smoothly maps Kadanoff's beard onto Mandelbrot's beard, a problem whose solution has eluded earlier renormalization group approaches. (While it is usually not appreciated that Mandelbrot has a beard, nothing prevents us from supposing that he carries it under his arm. In any case, it is all in the book[9].) However, MP is not in the book, as the book contains very many pictures that are not smooth.

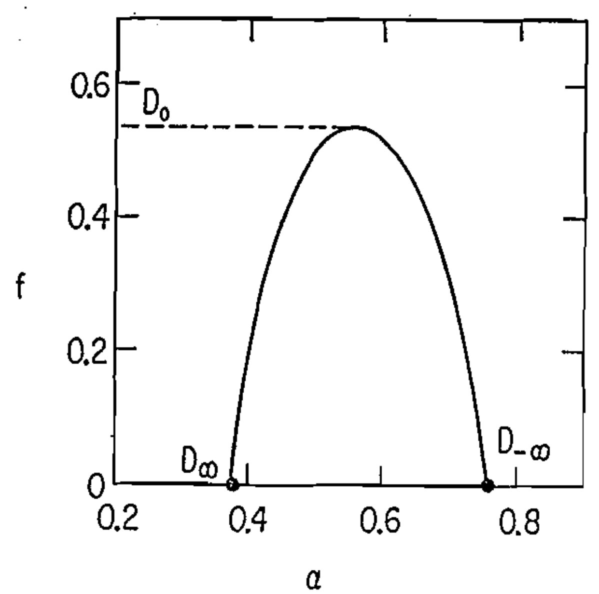

Fig. 2.3 A plot of f versus α for L.P. Kadanoff's beard.

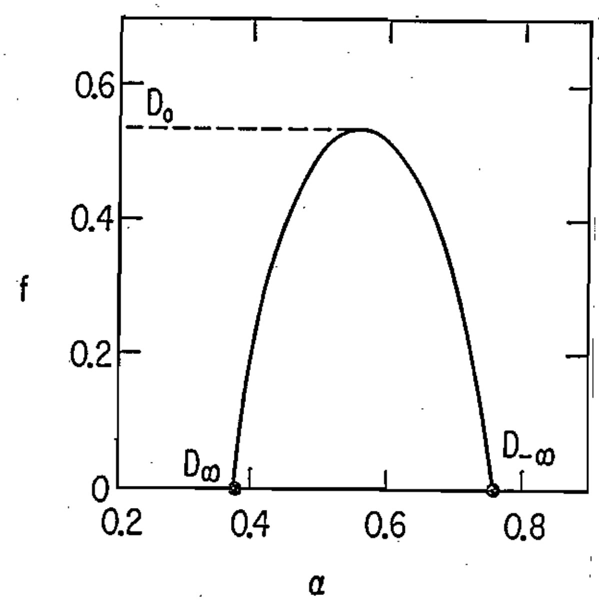

Fig. 2.4 A plot of f versus α for B.B. Mandelbrot's beard.

In our next example we return to turning Annette Wad into mashed potatoes. Note that the raw experimental contour, Fig. 1.3, to the eye does not appear to lie on a circle. Rather, it is twisted and contorted in a complicated way. Our results, Fig. 1.4, demonstrate however that from the metric point of view these two shapes <u>are the same</u> within experimental accuracy. To date we are not aware of any other approach that can lead to such a strong conclusion.

As our last example we use the alphabet. The alphabet is manifestly not smooth and not nice. The alphabet under MP is given in Fig. 2.5 and Fig. 2.6.

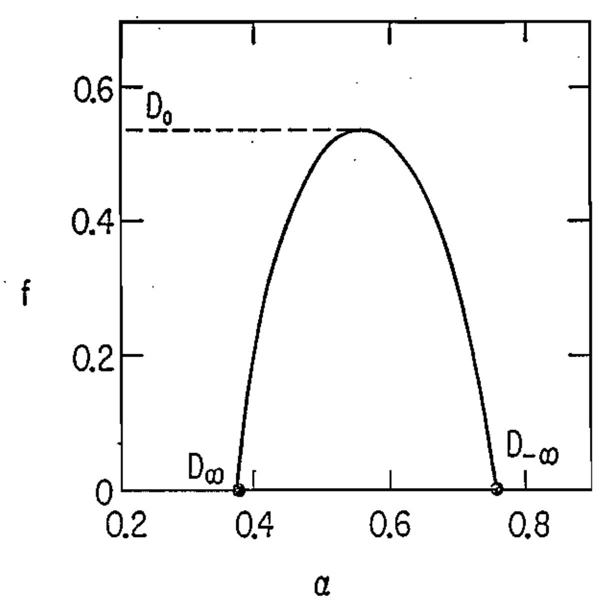

Fig. 2.5 A plot of f versus α for cirilic.

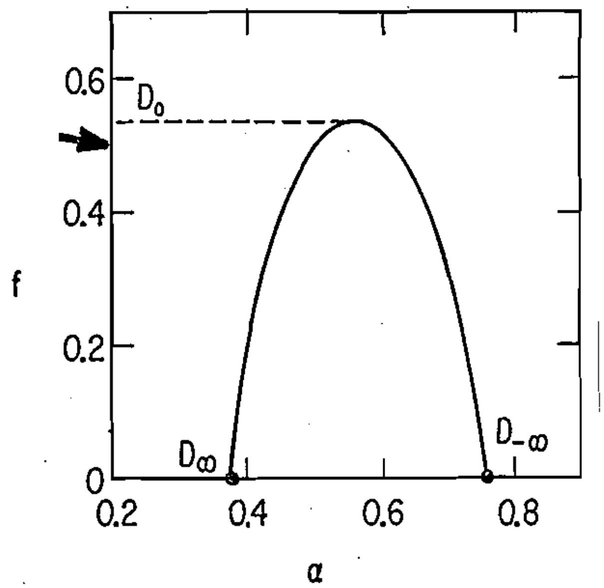

Fig. 2.6   A plot of f versus α for hebrew[10].

3 . CONCLUSION

Not only does this spectrum enrich our conceptual vocabulary - it has added the phrase MP to it.

In fact, MP is an ineffable paradigm, because it cannot be represented on a piece of paper: MP is perfectly smooth, but from the discussion leading up to Fig. 1.1 we know that a perfectly smooth paradigm cannot be represented on paper. As a matter of fact, and this is very deep - this brings us to the Godel's paradox - the very knowledge cannot be represented by books, as the paper is not smooth.

Thus the general shape of any curve for the description of truth is easy to grasp. Any problem reaches its solution through a precisely prescribed sequence of MP transformations. First, throw out the problem and replace it by a parabola. The second step is more subtle. You might imagine that you procede by succesivly magnifying the parabola. We have already learned that this leads to disaster. Instead, throw out the parabola and substitute it by any of the Figs 1.2 to 2.6.

The question of MP completness will be addressed in the forthcoming papers. 137 graduate students are already honing their wits in the pursuit of this profound question. If MP is really MP complete, then f can be smootly mapped into zero, with no loss of information.

With no pretense, this work represents a breakthrough of unparalelled import. In this connection ref. [11] might also be of interest.

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