
Problem 1

Do problem 9.3.1 from Strogatz.

I haven't formally introduceded the Lyapunov exponents in class yet, but you have already seen one of them when we discussed the exponential sensitivity of chaotic systems to initial conditions. Lyapunov exponents are the generalization of the Floquet exponents to aperiodic trajectories. The largest exponent λ_1 determines the average time rate of divergence of two trajectories, i.e.,

$$\lambda_1 = \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\vec{\eta}(t)|}{|\vec{\eta}(0)|},$$

where $\vec{\eta}(t) = \vec{x}_2(t) - \vec{x}_1(t)$ is an infinitesimal vector.

Problem 2

Do problem 9.3.8 from Strogatz.

Problem 3

To illustrate the "time horizon" after which the prediction for chaotic dynamics becomes impossible, numerically integrate the Rossler system

$$\dot{x} = -y - z, \qquad \dot{y} = x + ay, \qquad \dot{z} = b + z(x - c)$$

for $a = b = 0.2$ and $c = 5$.

(a) Starting at two initial conditions separated by a vector $\vec{\eta}(0)$ with an arbitrary direction and length 10^{-6}, compute and plot the separation $|\vec{\eta}(t)|$ between trajectories as a function of time. How long does it take for $\vec{\eta}(t)$ to become comparable to the size of the attractor?

(b) Repeat for an initial separation $\vec{\eta}(0)$ in the same direction, but with magnitude 10^{-12}.

Problem 4

Compute the orbit diagram for the Rossler system in the interval $2.5 \le c \le 4.5$ (you can again fix $a = b = 0.2$):

(a) Start at $c = 2.5$. Pick the initial condition in the vicinity of the unstable spiral fixed point and integrate the system forward, discarding the transient (a hundred or so time units). Plot some number of successive maxima of $x(t)$ to make sure that the system has converged to the limit cycle.

(b) Increase c slightly and continue the integration. (Do not restart the trajectory near the fixed point, doing so will make the transient longer!) After discarding the transient (ten or so time units) again plot some number of successive maxima of $x(t)$.

(c) Repeat (b) until you map the orbit diagram x vs. c. You have to plot at least 2^n successive maxima for a limit cycle produced after n period doubling bifurcations.

(d) Determine the values of c at which the first five period doubling bifurcations occur. You may want to take smaller steps in c near the bifurcations to refine your results.