Physics 136b

Homework associated with **Chapter 15,** Waves
Handed out Feb 7, 2001
Due, Feb 14

As usual, if any problem is trivial for you, do not do it – simply state that it is trivial and pick some other problem or make up your own. Problems 2.3 & 4 are completely new (i.e., not in the book), which means that it might have some ambiguity or error; contact me (djs@gps.caltech.edu) if you have a question or concern about it. Problems 2 & 4 require only order of magnitude estimates.

1. Ex. 15.3 (Ship Waves), B & T

2. Consider shallow water waves (wavelength long compared water depth).

(a)Show that the wave equation takes the form

$\partial^2\xi/\partial t^2 = g \quad .(D \quad \xi)$

where ξ is the vertical displacement of the free surface, g is gravitational acceleration and D is the depth of the fluid.
(b) Describe what happens to the direction of wave propagation as the depth of the ocean varies (either as a set of discrete jumps in D or as a slowly varying D). This is relevant to waves approaching a beach as well as the next part of the question. (But do not concern yourself with what happens when the wave is in such shallow water that non-linearities become important).
(c) Tsunamis are waves propagating on the deep ocean of Earth but are nonetheless shallow water waves because of wavelengths ~ 100km or so. Ocean depth is typically ~10km. What would you have to do to the ocean floor to create a lens that would focus tsunamis from Japan so as to destroy LA?

3. Consider waves propagating in a deformable, vertical, cylindrical conduit. The conduit has radius a(z,t) where z is the vertical coordinate (positive upwards) and is filled with liquid of density ρ_1. The conduit is

surrounded by a liquid of density $\rho_2 > \rho_1$. The conduit fluid has viscosity η_1 and the surrounding fluid has viscosity $\eta_2 >> \eta_1$.
(a) Confirm that to a good approximation,

$$Q = -\pi a^4[\, \partial p_1/\partial z + \rho_1 g]/8\eta_1$$

where Q is the volume flux of fluid-1 up the conduit, g is gravitational acceleration and a is assumed to vary slowly with height (long wavelength approximation).

(b) By considering normal stress balance across the wall of the conduit, show that to a good approximation

$$p_1 = -\rho_2 gz + 2(\eta_2 /a)\partial a/\partial t + \text{constant}$$

Hence write down the non-linear partial differential equation that involves Q and a. What is the other PDE relating Q and a?

(c) Consider solitary wave solutions of these equations which take the form $Q = Q_0 + Q_1 f(z-ct)$ and the corresponding form for a(z,t). Show that these waves satisfy the following relationship (relating wave amplitude to wave velocity):

$$c/v_0 = 2[\ln A - 1/2 + 1/2A^2]/[1-2/A + 1/A^2]$$

where v_0 is the background mean velocity in the undisturbed conduit (radius a_0) and $A=(a_{max}/a_0)^2$ is the wave amplitude. [Hint; You can integrate the resulting ODE once with respect to f. You do not need to solve the resulting non-linear ODE to derive the result].

These waves are discussed in Scott, Stevenson and Whitehead, *Nature* **319**, 759-761 (1986). They are almost solitons and are relevant to understanding porous flow and temporal fluctuations in volcanism of Hawaii (and perhaps even the sequence of islands that make up the Hawaiian island chain).

4. Seismic and acoustic wave evidence support the interpretation that an explosion was responsible for the recent (Russian) Kursk submarine disaster (August 12, 2000). A distinctive feature of these records is an

oscillation with frequency 1.45 Hz. Past experience with undersea explosions leads experts to interpret this as the "bubble pulse" which is oscillations of the rising bubble of hot gases produced by an explosion.

(a) Consider a bubble of gas surrounded by water at (nearly) the same pressure. Show that oscillations of the radius of the bubble have frequency

$$\omega = (c/a_0)\,(3\rho_g/\rho_0)^{1/2}$$

where c is the sound speed of the gas, a_0 is the equilibrium radius of the bubble, ρ_g is the density of the gas and ρ_0 is the density of the surrounding water. This is an acoustic oscillation since it involves the compressibility of the gas. However the "inertia" is dominated by the surrounding water.)

(b) The submarine was at depth ~100m at the time of the disaster. Using plausible parameters, estimate the size of the bubble and the work that must be done to create it. Compare this to the "expert" estimates of the required energy of the explosion, which give a few thousand kilograms of TNT equivalent (1 metric ton TNT ~ 4.2×10^{16} ergs.)

This is discussed in *EOS (Transactions American Geophysical Union)* **82,** p37 (Jan 23, 2001). Bubble pulse is discussed in , for example, *Waves in Fluids*, James Lighthill.