Physics 136b
(The instructor is now Dave Stevenson, through the remainder of this quarter)

Homework associated with **Chapter 14,** Turbulence

Handed out Jan 31, 2001
Due, Feb 7
To be graded by Alexander Putilin

As usual, if any problem is trivial for you, do not do it – simply state that it is trivial and pick some other problem or make up your own. Problems 1 & 2 are really a single problem with and without turbulence. Problem 4 is completely new (i.e., not in the book), which means that it might have some ambiguity or error; contact me (djs@gps.caltech.edu) if you have a question or concern about it. Problem 3 is the only one that requires any significant grunge; do one of the earlier problems in the same set if you've ever numerically integrated equations with this kind of behavior before.

1. Ex. 14.1, B & T
2. Ex 14.4, B & T
3. Ex 14.12, B & T

4. Earth has normal modes of oscillation, many of which are in the milliHertz frequency range. Large earthquakes occasionally but strongly excite these modes, but the quakes are usually widely spaced in time compared to the ring down time of a particular mode (which is typically a few days). However, there is now evidence that there is a background level of continuous excitation of these modes, so that there is typically around 10^{-10} cm/sec^2 rms ground acceleration per mode at seismically "quiet" times. Stochastic forcing by the pressure fluctuations associated with atmospheric turbulence is suspected. This question deals with some aspects of this hypothesis.

   (a) Barometric records at Earth's surface show that P(f) ~ 1/f, where P(f) is the characteristic pressure fluctuation at frequency f (you can think of P(f) as the rms pressure spectrum integrated over a frequency window of width f, and centered on f, so P has units of pressure). P(f = 1 mHz) ~ 0.5 Pa, which is about 5 x 10^{-4} of atmospheric pressure. Show that this power law is consistent with a Kolmogorov spectrum.
   (b) The low frequency cut-off for this pressure spectrum is about 0.5 mHz. Assuming this corresponds to the largest eddies which have a

length scale ~ few km (a little less than the scale height of the atmosphere), derive an estimate for the eddy viscosity (or, equivalently, diffusivity) of the lowermost atmosphere. By how many orders of magnitude does this exceed molecular values for diffusive transport? What fraction of the solar energy input (~10^6 erg/cm^2.sec) goes into maintaining this turbulence (assumed to be distributed over the lowermost 10km of the atmosphere only)?

(c) At 1mHz, what is the characteristic spatial scale of the relevant normal modes? The relevant modes have few or no nodes in the radial direction. All you need to answer this is a typical wave speed for seismic shear waves, which you can take to be 5km/sec. What is the characteristic spatial scale of the pressure fluctuations at the same frequency? (Assume isotropic turbulence). Suggest a plausible estimate for the rms amplitude of the pressure fluctuation averaged over a surface area equal to the mode wavelength squared. (You must keep in mind the spatially and temporally random fluctuating character of the turbulence.)

(d) Using your answer from (c), and a characteristic modulus for Earth deformation of ~ 10^{12} dynes/cm^2, comment on how the observed rms acceleration (10^{-10} cm/sec^2) compares with that expected by stochastic forcing due to turbulence. You may need to go back to earlier chapters (10 & 11) and think about the relationship between surface force and surface deformation. [Note: There are several issues in doing this assessment accurately that I've not spelled out (e.g. number of modes in a given frequency range); don't expect to be able to get an accurate answer.]

The background literature for this problem includes Tanimoto and Um, *J. Geophys. Res.* **104** 28723-28739 (1999). Related helioseismological classic reference is Goldreich and Keeley, *Astrophys. J.* **212** 243-251 (1977).