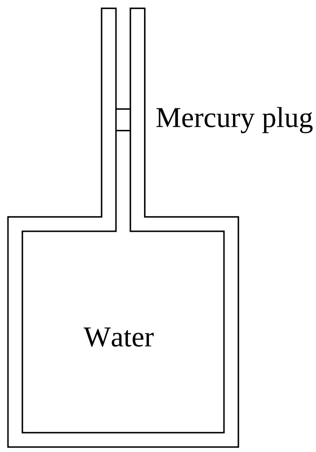
10346: Continuum Physics, spring 2002
Homework assignment 1

Measuring the compressibility of water

To measure the compressibility of water one uses a cylindrical glass container with thick walls (see figure), closed in one end and open in the other end through a long thin glass pipe. A mercury plug is mounted in the glass pipe, and the motion of the plug allows a precise determination of the volume changes of the water. We assume everywhere the validity of Hooke's law for isotropic materials.



The inner radius of the glass container is $a = 5$ cm, and the inner radius of the thin pipe is $c = 2$ mm. The inner height of the glass container is $L = 10$ cm. The bulk modulus for water is $K = 23$ kilobar.

We now apply the pressure $P = 100$ bar to the water through the mercury plug. We assume that the container is completely rigid and therefore unchanged by the increase of pressure.

1. Calculate the volume change of the water and the displacement of the mercury plug due to the pressure increase.

We now take into account that the glass container expands. The thickness of the glass walls is $d = 1$ cm. Young's modulus for the glass is $E = 750$ kilobar and Poisson's ratio is $\nu = 0.17$. We neglect the change in length of the container, assuming that it only expands radially.

2. Calculate the increase in the inner radius of the glass container.

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3. Calculate the displacement of the mercury plug in this case. Which percentage of this displacement is due to the expansion of the glass?

To avoid errors in his experiments in the 1820'ies, H. C. Ørsted enclosed the glass container in yet another container, so that the pressure was the same inside and outside the original glass container. Volume changes from the water in the glass container could still be measured by the motion of the mercury plug.

4. Find the radial displacement field in the glass container.

5. Calculate the change in the thickness of the glass container.

6. How big is the error in the determination of the change in volume (as measured by the mercury plug) in this case.

In fact, it is not necessary to assume that the glass container only expands radially.

7. Show that the deformation of the glass can be found exactly for any shape of the glass container. (*Hint:* Consider a deformation created by the scale transformation $\mathbf{x} \to s\mathbf{x}$ where s is a constant.) How does this influence the result in 6?

Due at the lecture *Monday, March 18.*

Have fun!