Physics 136b

Homework associated with **Chapter 16,** Supersonic Flow
Available Feb 19, 2001
Due, **Monday** Feb 26

As usual, if any problem is trivial for you, do not do it – simply state that it is trivial and pick some other problem or make up your own. Contact me (djs@gps.caltech.edu) if you have a question or concern about Problems 1 & 2 , which are new. There are only three problems this week.

1. If you place a gaseous (Jupiter-like) planet too close to a star then it will tend to "evaporate": A solar wind-like outflow will develop, driven by the UV flux of the star. This limits the survival time for some of the kinds of extrasolar planets recently discovered in very short period orbits.

   (a) Consider a steady, ideal gas outflow from a spherical gaseous planet. Show that the relevant set of equations (energy, momentum and continuity) can be written as:

$$\rho u \frac{d}{dr}\left(\frac{1}{2}u^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} - \frac{GM}{r}\right) = \Gamma(r)$$

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2}$$

$$\frac{dM}{dt} = -4\pi\rho(r)\,u(r)\,r^2$$

   where u is the radial gas flow, ρ is the gas density, p is the pressure, γ is the ratio of specific heats, M is the planetary mass (considered to be almost entirely below the radius of interest) and Γ is the volumetric heating rate (caused by the UV flux). Steady flow means dM/dt is a constant (on a dynamical time scale, not necessarily long time scales).

   (b) From these equations, derive an equation (of the standard solar wind or transonic form):

$$(u^2 - c^2)\frac{1}{u}\frac{du}{dr} = something$$

$$c^2 \equiv \frac{\gamma p}{\rho}$$

from which you should derive the conditions for a transonic flow (the only solution that crosses u < c to u>c as r increases):

$$c_s^2 = u_s^2 = \frac{GM}{2r_s} + \frac{(\gamma - 1)}{2}\left(\frac{\Gamma r}{\rho u}\right)_s$$

where subscript s means the quantity evaluated at the critical (sonic) radius r_s (the value of which is implicitly defined by this equation).

(c) From the energy equation and constant mass flux, derive the *approximate* result

$$\left|\frac{dM}{dt}\right| = \int_{r_m}^{r_s} 4\pi r^2 \Gamma(r)\, dr \,/\, \left[\frac{GM}{r_m} + \frac{5 - 3\gamma}{4(\gamma - 1)}\frac{GM}{r_s} + \frac{\gamma + 1}{4}\left(\frac{\Gamma r}{\rho u}\right)_s\right]$$

where subscript m refers to the smallest radius that the UV effectively penetrates (called optical depth unity in UV). This result neglects u_m^2 and p_m/ρ_m relative to GM/r_m, which is physically plausible except at very high fluxes.

(d) At $r > r_m$, we have $\Gamma \sim \kappa\rho F_{UV}$ where κ is the UV opacity and F_{UV} is the UV flux (this assumes strong efficiency of conversion of UV flux to heating, which is often true) . And we have the statement that defines r_m:

$$\int_{r_m}^{\infty} \rho\kappa\, dr \sim 1$$

Deduce that a plausible approximation is

$$\left|\frac{dM}{dt}\right| = \frac{4\pi r_m^3 F_{UV}}{GM}$$

and offer a simple physical interpretation of this result. (“Plausible” means you need to suggest why other terms might be neglected.)

(e) Find the survival time for a 10^{30}g body (half Jupiter mass) of radius 10^{10} cm in a UV flux of 4 x10^5 erg/cm^2.sec (corresponding to about 0.05 AU orbital radius). [In reality, these kinds of bodies actually get larger as mass is removed, which means they will eventually flow over the Roche lobe, defined as the region in which the planetary gravity dominates].

2. A spherical projectile of radius r and density ρ_o=3g/cm^3 enters Earth's atmosphere on a vertically downward trajectory, with initial velocity 15km/sec. The atmosphere is approximated to be isothermal: $\rho = \rho_g \exp(-z/H)$ where ρ_g =10^{-3}g/cm^3, z is height above the ground and H=10km is the atmospheric scale height. The pressure at z=0 is 1bar (10^6 dynes/cm^2). Since H is small compared to Earth's radius, the acceleration due to gravity is unimportant during passage through the atmosphere. You can treat the projectile as being deccelerated from the initial velocity by a drag force that can be estimated by considering the consequences of the shock front that forms just ahead of the front face of the projectile .

(a) Write down and justify an approximate equation of motion for the projectile. (Don't expect to be able to estimate the drag force to better than a factor of ~2).

(b) Solve the equation of motion (i.e. obtain the projectile velocity as a function of height above the ground).

(b) Show that if the projectile is small enough, the peak gas drag stress on the projectile occurs at some finite altitude rather than just before impact. Explain why small projectiles (e.g. meteorites the size of your fist or your head) can reach the ground intact while larger projectiles *made of the same material* (e.g. the body responsible for the Tunguska event in Siberia in 1908) will disrupt ("explode" in the air as a "fireball"). The stress needed for failure is typically in the 100-1000 bar range.

3. Exercise 16.9 (Stellar Winds) in the Book.