Problem 1

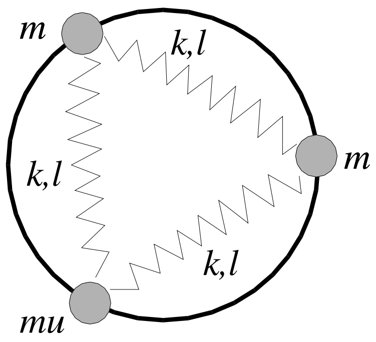
Consider the matrix

$$A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 2 & 1 \\ -1 & -1 & 1 \end{pmatrix}.$$

Determine whether the matrix is diagonalizable or not. Find the similarity transformation which converts the matrix into a diagonal or Jordan normal form.

Problem 2

Three (very small) beads with masses m, m, and μ can slide without friction on a (very thin) fixed ring of radius a and are connected to each other by (weightless) springs of equal stiffness k and equilibrium length $l = \sqrt{3}a$. Find the normal frequencies and normal modes of vibration for this system. Interpret the physical meaning of the modes you have found.



Problem 3

Find the solution to the ordinary differential equation

$$y'''' + 4y''' + 5y'' + 2y' = 0$$

subject to the initial conditions $y(0) = 1$, $y'(0) = -1$, $y''(0) = 2$, $y'''(0) = -5$ using the technique developed in class.

a) Rewrite this equation as a system of first order equations, $\vec{y}' = A\vec{y}$.

b) Find the eigenvalues and eigenvectors (or, failing that, generalized eigenvectors) of A.

c) Construct the Jordan normal form $\Lambda = S^{-1}AS$.

d) Solve the resulting system of equations in the new variables $\vec{u} = S^{-1}\vec{y}$.

e) Finally, use the backward transformation to find the solution $y(x)$.