
Problem 1

Evaluate the following integrals using contour integration:
a)

$$\int_{-\infty}^{\infty} \frac{e^{i\omega t} d\omega}{\omega^2 \pm \omega_0^2},$$

b)

$$\int_0^{2\pi} \frac{d\theta}{(a + b\cos\theta)^2}, \qquad (a > b > 0).$$

c)

$$\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} dx.$$

(Hint: $2\sin^2 x = 1 - \cos 2x$)

Problem 2

A function $f(z)$ is analytic along the real axis except for a third-order pole at $z = x_0$. The Laurent expansion about $z = x_0$ has the form

$$f(z) = \frac{a_{-3}}{(z - x_0)^3} + \frac{a_{-1}}{(z - x_0)} + g(z),$$

with $g(z)$ analytic everywhere. Show that the Cauchy principal value technique is applicable in the sense that
a)

$$\lim_{\delta \to 0} \left\{ \int_{-\infty}^{x_0 - \delta} f(x)dx + \int_{x_0 + \delta}^{\infty} f(x)dx \right\}$$

is well behaved.
b)

$$\int_{C_{x_0}} f(z)dz = \pm i\pi a_{-1},$$

where C_{x_0} denotes a small semicircle about $z = x_0$.