mathematical methods - week 14

Math for experimentalists

Georgia Tech PHYS-6124

Homework HW #14 due Monday, November 25, 2019

== show all your work for maximum credit,
== put labels, title, legends on any graphs
== acknowledge study group member, if collective effort
== if you are LaTeXing, here is the source code

Exercise 14.1 *A "study" of stress and life satisfaction* a) to d) 10 points

Bonus points
Exercise 14.1 *A "study" of stress and life satisfaction* e) 4 points
Exercise **??** *More to come:)* 4 points

Total of 10 points = 100 % score. Extra points accumulate, can help you later if you miss a few problems.

edited November 22, 2019

Week 14 syllabus

November 18, 2019

Lecturer: Ignacio Taboada

Mon Uncertainty, probability, probability density functions, error matrix, etc.

- Ignacio's lecture notes.
- Predrag's summary of key concepts for a physicist: ChaosBook Sect. 17.1.3 Moments, cumulants.

Wed Distributions: binomial; Poisson; normal; uniform. Moments; quantiles.

- Ignacio's lecture notes.
- For binomial theorem, Poisson and Gaussian distributions, see Arfken and Weber [1] (click here) *Mathematical Methods for Physicists: A Comprehensive Guide*, Chapter 19.

Fri Monte Carlo (why you need the uniform ditribution); central limit theorem (why you need normal dist). Multi-dimensional PDFs; correlation error propagation.

- Ignacio's lecture notes.
- Predrag's Noise is your friend and 33.3 Noisy trajectories derive the closely related covariance matrix evolution formula

14.1 Statistics for experimentalists: desiderata

I have solicited advice from my experimental colleagues. You tell me how to cover this in less than two semesters :)

2012-09-24 Ignacio Taboada Cover least squares. To me, this is the absolute most basic thing you need to know about data fitting - and usually I use more advanced methods.

For a few things that particle and astroparticle people do often for hypothesis testing, read Li and Ma [3], *Analysis methods for results in gamma-ray astronomy*, and Feldman and Cousins [2] *Unified approach to the classical statistical analysis of small signals*. Both papers are too advanced to cover in this course, but the idea of hypothesis testing can be studied in simpler cases.

2012-09-24 Peter Dimon thoughts on how to teach math methods needed by experimentlists:

1. Probability theory
   (a) Inference
   (b) random walks
   (c) Conditional probability
   (d) Bayes rule (another look at diffusion)

(e) Machlup has a classic paper on analysing simple on-off random spectrum. Hand out to students. (no Baysians use of information that you do not have) (Peter takes a dim view)

2. Fourier transforms
3. power spectrum - Wiener-Kitchen for correlation function
   (a) works for stationary system
   (b) useless on drifting system (tail can be due to drift only)
   (c) must check whether the data is stationary
4. measure: power spectrum, work in Fourier space
   (a) do this always in the lab
5. power spectra for processes: Brownian motion,
   (a) Langevin $\rightarrow$ get Lorenzian
   (b) connect to diffusion equation
6. they need to know:
   (a) need to know contour integral to get from Langevin power spectrum, to the correlation function
7. why is power spectrum Lorenzian - look at the tail $1/\omega^2$
   (a) because the cusp at small times that gives the tails
   (b) flat spectrum at origin gives long time lack of correlation
8. position is not stationary
   (a) diffusion
9. Green's function
   (a) δ fct $\rightarrow$ Gaussian + additivity
10. Nayquist theorem
    (a) sampling up to a Nayquist theorem (easy to prove)
11. Other processes:
    (a) what signal you expect for a given process
12. Fluctuation-dissipation theorem
    (a) connection to response function (lots of them measure that)
    (b) for Brownian motion power spectrum related to imaginary part of response function
13. Use *Numerical Recipes* (stupid on correlation functions)
    (a) zillion filters (murky subject)
    (b) Kalman (?)
14. (last 3 lecturs)
    (a) how to make a measurement
    (b) finite time sampling rates (be intelligent about it)

PS: Did I suggest all that? I thought I mentioned, like, three things.

Did you do the diffusion equation? It's an easy example for PDEs, Green's function, etc. And it has an unphysically infinite speed of information, so you can add a wave term to make it finite. This is called the Telegraph Equation (it was originally used to describe damping in transmission lines).

What about Navier-Stokes? There is a really cool exact solution (stationary) in two-dimensions called Jeffery-Hamel flow that involves elliptic functions and has a symmetry-breaking. (It's outlined in Landau and Lifshitz, *Fluid Dynamics*).

2012-09-24 Mike Schatz .

1. 1D bare minimum:
   (a) temporal signal, time series analysis
   (b) discrete Fourier transform, FFT in 1 and 2D - exercises
   (c) make finite set periodic
2. Image processing:
   (a) Fourier transforms, correlations,
   (b) convolution, particle tracking
3. PDEs in 2D (Matlab): will give it to Predrag

References

[1] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists: A Comprehensive Guide*, 6th ed. (Academic, New York, 2005).

[2] G. J. Feldman and R. D. Cousins, "Unified approach to the classical statistical analysis of small signals", Phys. Rev. D **57**, 3873–3889 (1998).

[3] T.-P. Li and Y.-Q. Ma, "Analysis methods for results in gamma-ray astronomy", Astrophys. J. **272**, 317–324 (1983).

Exercises

14.1. **A "study" of stress and life satisfaction.**

Participants completed a measure on how stressed they were feeling (on a 1 to 30 scale) and a measure of how satisfied they felt with their lives (measured on a 1 to 10 scale). Participants' scores are given in table 14.1.

You can do this homework with pencil and paper, in Excel, Python, whatever:

a) Calculate the average stress and satisfaction.

Participant	Stress score (X)	Life Satisfaction (Y)
1	11	7
2	25	1
3	19	4
4	7	9
5	23	2
6	6	8
7	11	8
8	22	3
9	25	3
10	10	6

Table 14.1: Stress vs. satisfaction for a sample of 10 individuals.

b) Calculate the variance of each.

c) Plot Y vs. X.

d) Calculate the correlation coefficient matrix and indicate the value of the covariance.

e) Bonus: Read the article on "The Economist" (if you can get past the paywall), or, more seriously, D. Kahneman and A. Deaton -the 2002 Nobel Memorial Prize in Economic Sciences- about the correlation between income and happiness. Report on your conclusions.