Chapter 19 Why cycle?

Solution 19.3:

(d) *In the $A = 9/2$ case all cycles up to length 9 yield $\lambda = 1.08569\ldots$. (Vadim Moroz)*

Solution 14.4: *The escape rate is the leading zero of the zeta function*

$$0 = 1/\zeta(\gamma) = 1 - e^{\gamma}/2a - e^{\gamma}/2a = 1 - e^{\gamma}/a.$$

So, $\gamma = \log(a)$ if $a > a_c = 1$ and $\gamma = 0$ otherwise. For $a \approx a_c$ the escape rate behaves like

$$\gamma(a) \approx (a - a_c).$$

Solution 19.1: *The escape is controlled by the size of the primary hole of the repeller. All subholes in the repeller will be proportional with the main hole. The size of the main hole is $l = \sqrt{1 - 1/a}$. Near $a_c = 1$ the escape rate is*

$$\gamma(a) \sim (a - a_c)^{1/2}.$$

We can generalize this and the previous result and conclude that

$$\gamma(a) \sim (a - a_c)^{1/z},$$

where z is the order of the maximum of the single humped map.

Solution 19.2: *By direct evaluation we can calculate the zeta functions and the Fredholm determinant of this map. The zeta functions are*

$$1/\zeta_k(z) = \det(1 - z\mathbf{T}_k),$$

where

$$\mathbf{T}_k = \begin{pmatrix} T_{00}^{k+1} & T_{01}^{k+1} \\ T_{10}^{k+1} & T_{11}^{k+1} \end{pmatrix},$$

and $T_{00} = 1/a_1$, $T_{01} = (b - b/a_1)/(1 - b)$, $T_{11} = (1 - b - b/a_2)/(1 - b)$, $T_{10} = 1/a_2$ are inverses of the slopes of the map. The Fredholm determinant is the product of zeta functions

$$F(z) = \prod_{k=0}^{\infty} 1/\zeta_k(z).$$

The leading zeroes of the Fredholm determinant can come from the zeroes of the leading zeta functions.

The zeroes of $1/\zeta_0(z)$ are

$$\begin{aligned} 1/z_1 &= \frac{T_{00}+T_{11}+\sqrt{(T_{00}-T_{11})^2+4T_{01}T_{10}}}{2}, \\ 1/z_2 &= \frac{T_{00}+T_{11}-\sqrt{(T_{00}-T_{11})^2+4T_{01}T_{10}}}{2}. \end{aligned}$$

The zeroes of $1/\zeta_1(z)$ are

$$\begin{aligned} 1/z_3 &= \frac{T_{00}^2+T_{11}^2+\sqrt{(T_{00}^2-T_{11}^2)^2+4T_{01}^2T_{10}^2}}{2}, \\ 1/z_4 &= \frac{T_{00}^2+T_{11}^2-\sqrt{(T_{00}^2-T_{11}^2)^2+4T_{01}^2T_{10}^2}}{2}. \end{aligned}$$

By substituting the slopes we can show that $z_1 = 1$ is the leading eigenvalue. The next to leading eigenvalue, which is the correlation decay in discrete time, can be $1/z_3$ or $1/z_2$.