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# Localized Control of Spatiotemporal Chaos

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## 1.1
## Introduction

Many physical, chemical, and biological systems of interest evolve in a non-equilibrium environment. As these systems are driven further out of equilibrium, they tend to display progressively more complicated dynamics, with steady spatially uniform states replaced first by non-chaotic patterned states and eventually by spatiotemporal chaos. This complexity is often undesirable and considerable benefits could be derived by forcing the system towards a less complex (but usually unstable) steady or time-periodic state. In response to this challenge, control of spatiotemporal chaos has emerged in recent years as a problem of increasing fundamental and applied value.

Control of turbulent boundary flows [17], mechanical vibrations, and noise [37] is already an indispensable component of industrial design. Many other significant technological applications, such as mixing [62], optical fiber manufacture [70], coating [4, 39], wide aperture semiconductor lasers [56], inertial confinement [67], combustion [81], and chemical reactions [10], could crucially benefit from our ability to control (either suppress or enhance) the instabilities leading to complex spatiotemporal dynamics. Considerable effort is currently being invested in control of ventricular fibrillation [79] and epilepsy [23].

Besides these practical applications, the ability to control spatiotemporal dynamics opens up a whole new direction in fundamental research by providing a unique capability to study otherwise inaccessible unstable states of extended non-equilibrium systems. This capability can be used, for instance, to experimentally construct complete bifurcation diagrams [49], study the dynamics and stability of isolated modes [22], detect and study unstable recurrent patterns [5], or reproducibly impose initial conditions [74].

Although the first attempts to control spatiotemporally complex dynamics are centuries old, a scientific approach has not been employed until 1904, when boundary layer theory was developed by Prandtl [68]. Subsequent at-

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tempts to suppress turbulence, from either empirical or linear stability perspective, lead to the creation of the field of flow control. More recently, control of low-dimensional chaos in nonlinear systems, that originated with the work of Ott, Grebogi, and Yorke [61] based on Floquet theory, has been extended to spatiotemporal dynamics. These two, originally independent, tracks have now merged, as recent studies (e.g., by Kawahara [47]) indicate.

The following classification [17] of various control approaches is helpful: By *passive* control we will understand applying any time-independent perturbation which tends to suppress the instability. *Predetermined active* control goes one step further by generalizing the class of perturbations to include time-dependence. We will refer to these two approaches as *open-loop* control. In contrast, the *closed-loop* control is based on feedback: it aims to stabilize an unstable steady (or time-periodic) state of the system by applying perturbations which depend on the deviation from that state.

Below, we will concentrate on closed-loop control as the most advanced way to influence the dynamics. Although it is more difficult to design and implement, closed-loop control offers a number of significant advantages over the open-loop variety. First of all, closed-loop control can be systematically designed by following a few rather general principles, while no systematic ways of designing open-loop control exist. Closed-loop control is substantially more energy-efficient because the magnitude of feedback depends on the deviation from the target state: effectively the control is switched off in the absence of disturbances, while the open-loop control is always on. Equally important from the practical standpoint, closed-loop control is generally more flexible and robust: it can be designed to handle noise and uncertainties in the modeling and parameters. From the fundamental perspective, closed-loop control provides a unique capability to study unstable states inaccessible without control by changing their stability properties, in contrast to open-loop control which replaces the unstable states with *different* stable states.

Although the field of closed-loop control of complex systems is much younger than the field of flow control (its roots can be traced to work by Lions on optimal control of systems governed by PDEs [54] in early 1970s) it too has reached a certain level of maturity. This can be attested by a number of successful experimental implementations in systems such as vibrating beams [13], chemical reactions [65], patches of heart tissue [21], plasma drift waves [24], and fluid convection in confined geometries [66, 76], all of which, in the absence of control, display *temporal* instabilities, but have a rather regular *spatial* structure. Most of these examples use the technique of single-input single-output (SISO) control, which is based on reconstructing the state of the system by making repetitive measurements of a single variable and then stabilizing one of the originally unstable steady or time-periodic states using a sequence of perturbations of a single parameter of that system.

This approach aimed at low-dimensional (due to strong geometrical confinement) systems breaks down, partially or completely, when applied to weakly confined systems, whose dynamics is characterized by spatial disorder and is, therefore, high-dimensional. The dynamics of such systems is rather weakly dependent on the boundary conditions and, as a result, recovers the symmetries inherited from the translational and rotational invariance of the unbounded physical space. These symmetries lead to degeneracies in the evolution operators describing the dynamics near the target state, resulting in the failure of single-parameter control. As previous studies (e.g., [26]) have shown, these degeneracies require the use of multi-point, or distributed, measurements (sensing) and feedback (actuation), i.e., multiple-input multiple-output (MIMO) control.

### 1.1.1 Empirical control

Several empirical methods have been developed to achieve control of weakly confined systems without relying on the knowledge on the evolution equations. Their complete description can be found elsewhere in this book. The simplest one is a generalization of Pyragas' time-delay autosynchronization (TDAS) algorithm [69] and can be used to stabilize an unstable *time-periodic* orbit (e.g., a plane wave) with temporal period $\tau$ by applying feedback proportional to the difference in the state of the system at times $t$ and $t-\tau$. The extended version of this algorithm (ETDAS) suggested by Bleich and Socolar [8], constructs the feedback as a weighted difference between the states of the system as times $t,\ t-\tau,\ t-2\tau,\ \cdots$. (E)TDAS was found to have a limitation when applied to spatially extended system in more than one spatial dimension, though: control fails for the target states with an odd number of unstable eigenvalues [46,58], the origin of which can be traced to spatial symmetries [38]. The generalized version of ETDAS (GETDAS) [57] goes around this limitation by replacing scalar weights with matrices. Yet, even GETDAS fails if there are stationary modes in the uncontrolled system [80], as is the case in weakly confined systems with continuous translational and/or rotational symmetry, characterized by the existence of symmetry-related Goldstone modes with zero growth rates.

Other studies have shown the possibility to achieve control of unstable *steady* or *time-periodic* patterns by applying feedback proportional to the instantaneous local deviation of the system from the target state. This method is sometimes referred to as (local) proportional control (LPC) [40] or, in the context of fluid flows, opposition control [33]. Although this method is relatively simple and, like ETDAS, does not require the knowledge of the dynamical equations, it requires feedback to be applied to all degrees of freedom of the system (e.g., all components of the velocity vector at every point in space for fluid flows [31]) to be successful – a requirement that is essentially impossible to satisfy in practice. If LPC is applied only to selected degrees of freedom (e.g, to one component of the velocity everywhere in space [30] or to all components of the velocity along a boundary [11]) the spatiotemporal chaos cannot be completely suppressed.

### 1.1.2 Model-based control

If the equations governing the dynamics of the system are known, one can improve on these empirical control methods. In the model-based approach, the description of the system is usually simplified by collapsing it along the strongly confined spatial direction(s)[2] (say, $z$) using either mode truncation (e.g., as in the analysis of Rayleigh-Bénard convection (RBC) [41,76] or wide aperture lasers [6]) or an approximation based on a perturbation expansion (e.g., as in the lubrication, or long-wave length, approximation for thin film flows [27,60]), producing a reduced order model depending only on the extended (weakly confined) directions (say, $x$ and $y$). It is then assumed that spatially distributed feedback is applied by perturbing the system at all points $(x, y)$ in the extended directions by an amount proportional to the deviation of the system from the target state (either a uniform state or a plane wave) at the same location $(x, y)$. As a result, a system

$$\partial_t \mathbf{v} = \mathbf{f}[\mathbf{v}, u], \tag{1.1a}$$

$$u = kw, \qquad w = \mathbf{c} \cdot (\mathbf{v} - \mathbf{v}_0), \tag{1.1b}$$

is obtained, where $\mathbf{f}[\cdot, \cdot]$ and $\mathbf{v}(x, y, t)$ are, respectively, the evolution operator and the state of the open-loop system, $u(x, y, t)$ is the feedback (i.e., the disturbance applied to one of the system parameters), $\mathbf{c}$ is a constant vector describing the relation between the system state and the measurement $w(x, y, t)$, and the feedback gain $k$ is the proportionality constant between the deviation from the target state $\mathbf{v}_0(x, y, t)$ and the feedback signal.

In the physical system the deviation can often be measured at one (or both) of the boundaries, say $z = 0$ and $z = l_z$ (e.g., oxygen concentration on the surface of platinum catalyst in CO oxidation [63]), in a plane $z = z_0$ between the boundaries (e.g., velocity for a turbulent shear flow [51] or temperature for RBC [77]), or an integrated deviation for $0 < z < l_z$ can be used (e.g.,

**2**) In non-equilibrium systems, confined directions usually correspond to the direction of the flux driving the system out of equilibrium (e.g, momentum transport in shear fluid flows, heat flux in convection, etc.), while there is no flux, on average, in the extended directions [12]. Of course, it is possible that, in large aspect ratio systems, zero-mean-flux directions can effectively become confined as well.

shadowgraphic amplitude for RBC [42]). Similarly, feedback can be applied by changing the boundary conditions at one (or both) of the boundaries (e.g., heat flux through the boundary in RBC [42, 77]) or by applying an integrated disturbance (e.g., volumetric heating of the fluid in Marangoni-Bénard convection [74] or superimposition of the electromagnetic field with its filtered and time-delayed version in a He-Ne laser [53]). Since the closed-loop system is translationally invariant in the extended directions, the eigenfunctions of the reduced order model are given by either Fourier modes (for spatially uniform) or by Bloch-Floquet waves (for plane wave target states). Hence the linearized evolution equations block-diagonalize in the Fourier space, producing an infinite set of ODEs (state-space representation)

$$\begin{aligned} \dot{\bar{\mathbf{v}}} &= A\bar{\mathbf{v}} + \mathbf{b}\bar{u}, \\ \bar{u} &= k\mathbf{c} \cdot \bar{\mathbf{v}}, \end{aligned} \tag{1.2}$$

labeled by the wave number $\mathbf{q} = (q_x, q_y)$, where $\bar{\mathbf{v}}(\mathbf{q}, t) = \mathcal{F}_{\mathbf{q}}(\mathbf{v} - \mathbf{v}_0)$ and $\bar{u}(\mathbf{q}, t) = \mathcal{F}_{\mathbf{q}} u$ are the Fourier transformed state and feedback variables, $A(\mathbf{q}, t) = \mathcal{F}_{\mathbf{q}}(\partial \mathbf{f}/\partial \mathbf{v}|_{\mathbf{v}_0})\mathcal{F}_{\mathbf{q}}^{-1}$ is the Jacobian of the open-loop system and the vector $\mathbf{b}(\mathbf{q}, t) = \mathcal{F}_{\mathbf{q}}(\partial \mathbf{f}/\partial u|_{\mathbf{v}_0})\mathcal{F}_{\mathbf{q}}^{-1}$ describes how the feedback affects different degrees of freedom of the system. Finally, the feedback gain $k$ is chosen to simultaneously stabilize all Fourier modes.

Although this physically motivated approach often works well, it too has limitations. For instance, a constant gain $k$ stabilizing all Fourier modes might not exist, as, e.g., the analysis of the complex Ginzburg-Landau equation (CGLE) [7] and lubrication equations describing evaporating liquid films [27] shows. An improved version of this approach developed by one of us (RG) [27] and Bamieh *et al.* [3] uses the results of linear stability analysis to systematically design the feedback. The systematic approach shows that a stabilizing feedback can only be found when $A$, $\mathbf{b}$, and $\mathbf{c}$ satisfy certain restrictive conditions. These conditions are often (but certainly not always) satisfied for systems described by only a few coupled scalar fields. For instance, thin liquid films can be described by one variable (e.g., film height [60]), RBC requires two variables (e.g., temperature and vertical velocity [77]), while single mode wide-aperture laser models are three-dimensional (e.g., complex amplitude of the electric field and carrier density [6]).

When a stabilizing feedback does exist, it often has to be wave number-dependent (as well as time-dependent for time-periodic target states). Furthermore, optimal (in the sense of time-averaged deviation of the system from the target state) feedback [3, 27] is generically wave number-dependent and, therefore, nonlocal in the real space,

$$u = \int d\mathbf{q} \mathcal{F}_{\mathbf{q}}^{-1} k(\mathbf{q}) \mathcal{F}_{\mathbf{q}} w, \tag{1.3}$$

such that the feedback at a particular spatial location depends on the deviation from the target state at other locations. Several other theoretical [52] and experimental [43,55] studies of nonlinear optical systems have also found that Fourier filtered feedback is required in order to stabilize unstable patterns.

The model-based approach becomes indispensable when $A$, $\mathbf{b}$, and $\mathbf{c}$ do not satisfy the restrictions alluded to above, which is the generic case. This requires modifications to (1.1b) and (1.2) with the goal of reconstructing the deviation $\mathbf{v} - \mathbf{v}_0$ from the scalar measurement $w$ (see e.g., [14] for details). Specifically, (1.2) is replaced with

$$\begin{aligned} \dot{\bar{\mathbf{v}}} &= A\bar{\mathbf{v}} + \mathbf{b}\bar{u}, \\ \dot{\hat{\mathbf{v}}} &= A\hat{\mathbf{v}} + \mathbf{b}\bar{u} - \hat{\mathbf{k}}(\bar{w} - \mathbf{c}\cdot\hat{\mathbf{v}}), \\ \bar{u} &= \mathbf{k}\cdot\hat{\mathbf{v}}, \end{aligned} \tag{1.4}$$

where $\bar{w}(\mathbf{q}, t) = \mathcal{F}_{\mathbf{q}} w$, $\hat{\mathbf{k}}$ is the filter gain, and both it and the feedback gain $\mathbf{k}$ become vectors. Respectively, the first equation in (1.1b) is replaced with $u = \mathcal{F}_{\mathbf{q}}^{-1}\bar{u}$. By subtracting the second equation in (1.4) from the first, one finds that $\hat{\mathbf{v}} \to \bar{\mathbf{v}}$ provided $A + \hat{\mathbf{k}}\mathbf{c}^\dagger$ (or $A^\dagger + \mathbf{c}\hat{\mathbf{k}}^\dagger$) is stable, while $\bar{\mathbf{v}} \to 0$ provided $A + \mathbf{b}\mathbf{k}^\dagger$ is stable. We find that mathematically the problem of finding $\hat{\mathbf{k}}$ given $\mathbf{c}$ is equivalent to that of finding $\mathbf{k}$ given $\mathbf{b}$. It is a standard control-theoretic result that the feedback gain $\mathbf{k}$ and the filter gain $\hat{\mathbf{k}}$ can be found provided $A$ and $\mathbf{b}$ satisfy the *controllability* (or the weaker *stabilizability*) condition and $A$ and $\mathbf{c}$ satisfy the *observability* (or the weaker *detectability*) condition [14, 26]. This duality between the feedback and sensing parts of the controller allows one to solve both the problem of state reconstruction and the problem of feedback control using the assumption that the complete knowledge about the state of the system is available (i.e., replacing $\mathbf{c}$ with a unit matrix in (1.1b)). A review by Kim [48] discusses the application of this approach to control of turbulent boundary flows.

The model-based approach makes no assumptions regarding the number of unstable directions have to be made and access to all degrees of freedom (for sensing or actuation) is not required. This makes physical sense: for instance, in convective systems temperature perturbations also control the velocity, while in lasers the perturbations of the electric field also control the polarization and the population inversion. However, the crucial step in the analysis – block-diagonalization of the linearized evolution equations – hinges on the implicit assumption of translational (or rotational) invariance, which cannot always be justified.

For instance, the assumption that both sensing and feedback are spatially continuous (in other words, independent sensing is done, or feedback applied, at every point in the space of extended directions) is usually unrealistic, although the development of micro-electro-mechanical systems (MEMS) could change that. So far, systems for which both sensing and actuation can be done optically represent the only exception. For instance, all-optical analog feedback loops have proved effective for control of pattern formation in nonlinear optical systems [43,53,55,64], while the applicability of thermalized optical perturbations for control of thin film flows has been demonstrated by Semwogerere and Schatz [74] and Garnier *et al.* [22].

More typically, both sensing and feedback have to be implemented using an array of discrete elements. Several theoretical studies of coupled ODEs [59], CGLE [9, 45], and two-dimensional turbulence [31, 75] suggest that it is possible to achieve control using LPC applied via an array of spatially localized sensors and actuators, but that array should be rather dense. The existing convection experiments achieved *partial* stabilization of the flow by using a large number of small heaters (15 in Ref. [41], 24 in Refs. [76] and [49]), but provided little information on the relation between spatial resolution and the degree of stabilization. This relation, especially in the limit of sparse sensor/actuator arrays, is of fundamental importance from both physical and control theoretic perspective. Several different conjectures have been made regarding the density of the sensor/actuator array necessary to achieve control. Some studies suggest that the distance between the closest elements is determined by the correlation length [9, 45], while others suggest that the number of elements in the array should equal the number of unstable modes [1, 32]. Other studies [15,25,28] have shown that, for an appropriately chosen feedback, a much smaller density (limited by noise) of sensors/actuators can be achieved.

In the following sections we will discuss the conditions affecting the density and structure of the sensor/actuator array and describe how a stabilizing feedback gain can be computed. Although the generalization of our results to time-periodic target states is, in principle, straightforward, we will limit our discussion to steady states to make it more accessible. Furthermore, we will initially assume that complete information about the system state is available and then discuss how the results change if only partial information can be obtained using an array of sensors. Finally, we will assume that the system has only one extended direction (say, $x$) and is laterally bounded, $0 < x < l_x$ (we will drop the index of $l_x$ below).

## 1.2
## Symmetry and the minimal number of sensors/actuators

If feedback is applied via spatially localized actuators, how many such actuators, at a minimum, are needed to suppress chaos in favor of a particular target state? As the theory developed in [26] shows, the answer to this question depends on the symmetries of the system and the target state, but not on the system size or on how the feedback is computed. This is a fundamental issue that has to be understood before moving on.

In a laterally bounded system the wave numbers will be discrete rather than continuous, $q_x = \cdots, q_{-2}, q_{-1}, q_0, q_1, q_2, \cdots$. Defining the feedback signal applied by actuators $m = 1, 2, \cdots, M$ as $\mathbf{u} = (u_1, u_2, \cdots, u_M)$, we can write the evolution equations describing our system as

$$\dot{\bar{\mathbf{v}}}_n = A_n \bar{\mathbf{v}}_n + B_n \mathbf{u}, \qquad n = \cdots, -2, -1, 0, 1, 2, \cdots, \tag{1.5}$$

where $B(q) = \mathcal{F}_q(\partial \mathbf{f}/\partial \mathbf{u}|_{\mathbf{v}_0})$, $B_n = B(q_n)$, $A_n = A(q_n)$ and $\bar{\mathbf{v}}_n(t) = \bar{\mathbf{v}}(q_n, t)$. If we denote the number of scalar fields describing the state of the system (i.e., the dimensionality of $\mathbf{v}$) as $N$, then $A_n$ would be an $N \times N$ matrix and $B_n$ would be an $N \times M$ matrix.

Let $\beta_k$ be the eigenvalues of the full block-diagonal Jacobian of the system

$$\hat{A} = \mathrm{diag}(\cdots, A_{-2}, A_{-1}, A_0, A_1, A_2, \cdots) \tag{1.6}$$

and let $\mu_k$ be the degeneracy of eigenvalue $\beta_k$. Further, let

$$\bar{A}_k = \begin{pmatrix} A_{n_1} & & \\ & \ddots & \\ & & A_{n_\mu} \end{pmatrix}, \qquad \bar{B}_k = \begin{pmatrix} B_{n_1} \\ \vdots \\ B_{n_\mu} \end{pmatrix}, \tag{1.7}$$

where the indices run over the values of $n$ for which $\beta_k$ is an eigenvalue of $A_n$. It can be shown then [26], that the feedback $\mathbf{u}$ stabilizing the system (1.5) exists, provided (i) the number $M$ of columns of $B$ (and hence of actuators) is no less than the highest degeneracy of the unstable eigenvalues,

$$M \geq \max_{\mathrm{Re}(\beta_k)>0} \mu_k, \tag{1.8}$$

and (ii) at least one of the columns of $\bar{B}_k$ is non-orthogonal to the adjoint eigenvectors of $\bar{A}_k$ for all $k$ with $\mathrm{Re}(\beta_k) > 0$.

The degeneracy is usually determined by the symmetries of the evolution equation and once these symmetries are identified, the situation usually simplifies considerably. Let us look at some examples. Consider the complex Ginzburg-Landau equation with $\delta$-localized feedback

$$\partial_t v = \epsilon v + (1 + ib)\partial_x^2 v - (1 + ic)|v|^2 v + (1 + id) \sum_{m=1}^{M} \delta(x - x_m) u_m \tag{1.9}$$

and periodic boundary conditions on a domain of length $l = 2\pi$ (such that $q_n = n$). Assuming $u_m$ to be real, linearizing (1.9) about the steady state $v_0 = 0$ and Fourier transforming, we obtain the evolution equations for the real and imaginary parts of $\bar{v}_n = \bar{r}_n + i\bar{s}_n$:

$$\begin{pmatrix} \dot{\bar{r}}_n \\ \dot{\bar{s}}_n \end{pmatrix} = A_n \begin{pmatrix} \bar{r}_n \\ \bar{s}_n \end{pmatrix} + B_n \mathbf{u}, \qquad A_n = \begin{pmatrix} \epsilon - n^2 & bn^2 \\ -bn^2 & \epsilon - n^2 \end{pmatrix},$$
$$B_n = \begin{pmatrix} \cos(nx_1) - d\sin(nx_1) & \cdots & \cos(nx_M) - d\sin(nx_M) \\ d\cos(nx_1) + \sin(nx_1) & \cdots & d\cos(nx_M) + \sin(nx_M) \end{pmatrix}. \tag{1.10}$$

The reflection symmetry of the evolution equation (1.9) and the target state $v_0 = 0$ has transpired in the degeneracy of the eigenvalues of the linearized system, $\beta^{\pm}_{-n} = \beta^{\pm}_n = \epsilon - n^2 \pm ibn^2$. We find that $\mu_n = 2$ for all $n \neq 0$ and, consequently, at least two actuators are needed to stabilize the chosen target state. This is a special case of the general result proved in [26]: the minimal number of independent feedback signals should be no less than the dimensionality of the largest irreducible representation of the isotropy subgroup $\mathcal{G}_{\bar{v}_0}$ of the system, which is defined as a set of all transformations with respect to which both the open-loop evolution equation (i.e., (1.9) with $\mathbf{u} = 0$) and the target state are invariant. In this particular case $\mathcal{G}_{\bar{v}_0} = O(2) \times U(1)$ (spatial translations and reflection plus the global phase symmetry $v \to e^{i\phi} v$) and its largest irreducible representation is two-dimensional.

The second lesson can be learned by considering part (ii) of the stabilizability condition. Without loss of generality we can pick the origin of the coordinate system such that $x_1 = 0$, so that

$$\bar{A}_n = \begin{pmatrix} A_{-n} & 0 \\ 0 & A_n \end{pmatrix}, \qquad \bar{B}_n = \begin{pmatrix} 1 & \cos(nx_2) + d\sin(nx_2) \\ d & d\cos(nx_2) - \sin(nx_2) \\ 1 & \cos(nx_2) - d\sin(nx_2) \\ d & d\cos(nx_2) + \sin(nx_2) \end{pmatrix}. \tag{1.11}$$

It is easy to check that $\mathbf{e}^\dagger = (1, i, -1, -i)$ is an adjoint eigenvector of $\bar{A}_n$ and the condition (ii) is not satisfied whenever $e^\dagger \bar{B}_n = 0$ (or $x_2 = \pi/n$). In other words, stabilizability is lost whenever an unstable eigenfunction of the system, e.g., $v_n = \sin(nx)$, has nodes at the locations of both actuators.

Similar conclusions can be drawn for a laterally infinite system with non-local coupling defined by an integral, as opposed to a differential, equation. Consider, for instance, the following evolution equation:

$$\partial_t v(x,t) = \epsilon v(x,t) + \int_{-\infty}^{\infty} e^{-\frac{(x-x')^2}{2\sigma^2}} v(x',t)dx - v^3(x,t) + \sum_{m=1}^{M} \delta(x - x_m) u_m(t). \tag{1.12}$$

After linearization about the trivial steady state $v_0 = 0$ and Fourier transformation (1.12) reduces to a set of ODEs

$$\dot{v}_q = \beta_q v_q + \sum_{m=1}^{M} e^{iqx_m} u_m, \tag{1.13}$$

with doubly degenerate eigenvalues

$$\beta_{\pm q} = \epsilon + \sqrt{2\pi}\sigma e^{-\frac{\sigma^2 q^2}{2}}. \tag{1.14}$$

Consequently, at least a pair of actuators is needed, and the spacing should satisfy condition (ii) with

$$\bar{A}_q = \begin{pmatrix} \beta_q & 0 \\ 0 & \beta_q \end{pmatrix}, \qquad \bar{B}_q = \begin{pmatrix} 1 & e^{-iqx_2} \\ 1 & e^{iqx_2} \end{pmatrix}, \tag{1.15}$$

which requires $qx_2 \neq \pi n$ for all integer $n$ and all $q$ such that $\beta_q > 0$. Defining $\lambda_{\min}$ to be the smallest unstable wave length, we can write the stabilizability conditions as $|x_1 - x_2| < \lambda_{\min}/2$.

Summing up, we can formulate the following rule of thumb for control of steady uniform states in translationally and reflectionally invariant one-dimensional systems: *At least a pair of actuators separated by less than half the wavelength of every unstable mode is necessary to achieve stabilization*. In higher dimensions more actuators will be needed, as determined by the respective symmetry group.

Introduction of mean flux in any of the lateral directions changes these results dramatically. For instance, adding a reflection symmetry-breaking term $(1 + ia)\partial_x v$ to the right hand side of (1.9) changes the eigenvalues to

$$\beta_n^{\pm} = \epsilon - an - n^2 \pm i(bn^2 - n), \tag{1.16}$$

removing the reflection-related degeneracy for all $n$, $\beta_{-n}^{\pm} \neq \beta_n^{\pm}$. Since now all $\mu_n = 1$, just one actuator may be sufficient (in one dimension). This reduction in the minimal number of actuators provides, at least to some extent, the explanation for the observation that if either mean flux is introduced [18] or if the actuators are not stationary, but move through the system (regularly [75] or randomly [20]), fewer of them are needed to suppress chaos. Indeed, the introduction of a term such as $\mathbf{a} \cdot \nabla v$ into the right hand side of the evolution equation (1.1a) is equivalent to changing the reference frame to the one moving with velocity $\mathbf{a}$, in which the actuators, previously stationary, move with velocity $-\mathbf{a}$.

We conclude this section with a few general remarks. The stricter controllability condition requires satisfaction of (i) and (ii) for all $k$, stable and unstable. We should note, however, that spatially extended systems with a continuous spatial variable cannot be made controllable as stable modes with arbitrarily small wave lengths exist, so condition (ii) is impossible to satisfy. Second, using the duality of feedback and sensing parts, we conclude that the same conditions (i) and (ii) apply to an array of sensors.

Needless to say, one should not expect the results for the minimal number of actuators (or sensors) to hold in practice for systems of arbitrary size. The main reason for this is that linear stability analysis only considers the dynamics of infinitesimal disturbances, while real disturbances always have a finite size. We will look at the effect of disturbances in the next sections.

## 1.3 Nonnormality and noise amplification

If the system can be made formally stabilizable, control can only fail as a result of failure of linear stability analysis when disturbances grow so large that nonlinear terms become non-negligible. To determine the dynamics of disturbances, however, we do need to define how the feedback is computed.

Again, to illustrate the main idea we will restrict our attention to a narrower class of spatially extended systems, following our earlier study [35]. Specifically, we will consider scalar translationally and reflection symmetric versions of (1.1a) in one spatial dimension with periodic boundary conditions. Examples of this class of systems include such model equations as the real Ginzburg-Landau, Kuramoto-Sivashinsky, and Swift-Hohenberg equations. These model equations describe the dynamics of generic, spatially extended systems close to several common types of bifurcations [12] and thus are of particular importance in the studies of spatiotemporal dynamics.

As in the previous section, we will assume that the feedback is applied through an array of spatially localized actuators. Linearizing about a steady uniform state we obtain the following equation for the deviation from the target state

$$\partial_t v(x,t) = \hat{A} v(x,t) + \sum_{m=1}^{M} b_m(x) u_m(t) \tag{1.17}$$

where $\hat{A}$ is a linear operator and $b_m(x)$ are the influence functions describing the location and spatial extent of each of the $M$ actuators. Assuming the state of the system can be obtained either by direct measurements or via a state reconstruction procedure described in section 1.1.2 we can express the feedback signals $u_m(t)$ as linear functions of the deviation

$$u_m(t) = \int_0^l k_m(x) v(x,t) dx, \tag{1.18}$$

where $k_m(x)$ is the feedback gain that should be chosen such that the uniform state is stabilized. The non-locality of this feedback law is the price one has to pay for the generality of this approach that will allow us to use a very sparse array of actuators. In contrast, local proportional control (e.g., $u_m(t) = kv(x_m, t)$ with $b_m(x) = \delta(x - x_m)$) generically breaks down as soon as the distance between actuators exceeds $\lambda_{\min}/2$ (see, e.g., [19]).

To simplify the problem of computing $M$ feedback gains we can use the symmetry of the problem by making the actuators identical (e.g., by setting $b_m(x) = b_0(x - x_m)$) and placing the controllers in a regular array, so the closed-loop system will retain a discrete translational symmetry (subgroup of continuous translational symmetry of the open-loop system). However, placing the actuators in a periodic array will make the Fourier mode with the period equal to twice the array spacing $s = l/M$ uncontrollable and lead to the loss of stabilizability as long as that mode is unstable. We thus find that in a periodic array one should choose $s < s_{\max} \equiv \lambda_{\min}/2$, so that the number of actuators scales with the system size, $M \geq 2l/\lambda_{\min}$.

A significantly smaller number of actuators will be needed, if a periodic array of *pairs* of actuators is used instead, with the spacing $s_1$ in the pair smaller than $s_{\max}$ and the spacing $s_2 = 2l/M$ between the pairs potentially much larger than $s_{\max}$. The resulting array will have a discrete translational symmetry ($x \to x + s_2$) and a reflection symmetry about the midpoint between any neighboring actuators. These symmetries dictate the following choice of influence functions:

$$b_m(x) = \begin{cases} b_0\left(x - (m - \Delta)\frac{l}{M}\right), & \text{m−odd,} \\ b_0\left(x - (m - 1 + \Delta)\frac{l}{M}\right), & \text{m−even,} \end{cases} \tag{1.19}$$

where we have defined $\Delta = s_1/s_2$. For instance, four actuators would be placed as two pairs, one pair at $x = (1 \pm \Delta)l/4$ and the other at $x = (3 \pm \Delta)l/4$. To preserve the symmetries of the closed-loop system we also choose the gains $k_m(x)$ as translated and reflected versions of each other, mirroring the choice (1.19) we have made for the influence functions, so that only a single unknown weight function $k_0(x)$ needs to be determined (also see [3]).

Fourier transforming the linearized evolution equation (1.17) and the feedback law (1.18) we obtain the system

$$\dot{\bar{v}}_n = \beta_n \bar{v}_n + \sum_{m=1}^{M} B_n^m \sum_{p=-\infty}^{\infty} K_{-s}^p \bar{v}_p \equiv (M\bar{v})_n, \tag{1.20}$$

where $\bar{v}_n$, $B_n^m$, and $K_n^m$ are the Fourier coefficients of $v(x,t)$, $b_m(x)$, and $k_m(x)$, respectively, $\beta_n = \beta(q_n)$ (with $q_n = 2\pi n/l$) are the eigenvalues of the linearized open-loop system, and $M$ is the Jacobian of the closed-loop system.

At this point it is appropriate to mention that the choice of the influence function $b_0(x)$, which is determined by the physical construction of the actuators, plays an important role in the control problem. For instance, the Fourier spectrum of $b_0(x)$ should contain all unstable modes; modes missing in the spectrum will be uncontrollable. On the other hand, if the spectrum contains stable modes as well, the feedback stabilizing the unstable modes of the open-loop system can destabilize some of the stable modes. This problem is referred to as control spillover [32]. As a result, despite the block-diagonalization of the open-loop system, the calculation of feedback has to include all modes that appear in the spectrum of $b_0(x)$, both unstable and stable ones.

It turns out that the Fourier coefficients of $k_0(x)$ can be found analytically in the limit of singularly localized influence functions, $b_0(x) = \delta(x)$, as a function of the eigenvalues $\beta'_n$ of the closed-loop system. This results in the so-called pole placement control. The details can be found in [35]. Here we will mention the main result: the largest Fourier coefficient, and with it the maximum of $k_0(x)$, scales exponentially with the length of the system divided by the number of controllers

$$k_{\max} \sim e^{\frac{l}{Ml_0}}, \tag{1.21}$$

where $l_0$ is a characteristic length which, to leading order in $l$, is given by

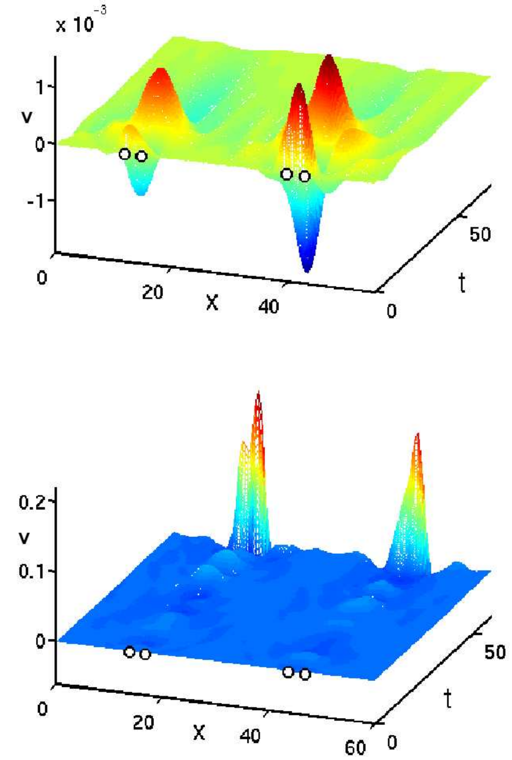
$$l_0 = \pi \left( \int_{-\infty}^{\infty} \ln \frac{|\beta_{\max} - \beta'(q)|}{|\beta_{\max} - \beta(q)|} dq \right)^{-1}, \qquad \beta_{\max} = \max_n \beta_n \tag{1.22}$$

(for large $l$ the wave numbers $q_n$ are dense, so we can parameterize new eigenvalues using a functional form $\beta'(q_n) = \beta'_n$).

This result shows that although in principle it is possible to find a stabilizing feedback for any system size $l$ and number of actuators $M$, the price one pays for making $l$ large or $M$ small is the exponential increase in the magnitude of the feedback signal applied by the actuators. It is not difficult to imagine the consequences of such a feedback: a small $O(\sigma)$ initial disturbance would generate an $O(k_{\max}\sigma)$ perturbation applied by the nearby actuators resulting, at least temporarily, in the amplification of that initial disturbance by roughly a factor of $k_{\max}$. Since the closed-loop system is linearly stable, all sufficiently small disturbances will eventually decay, making this feedback-generated disturbance amplification transient. Mathematically, transient growth of disturbances can be related to the nonnormality of the Jacobian $M$ of the closed-loop system and is characterized by the transient amplification factor

$$\gamma \equiv \max_{t,\bar{\mathbf{v}}(0)} \frac{\|\bar{\mathbf{v}}(t)\|_2}{\|\bar{\mathbf{v}}(0)\|_2} = \max_t \left\| e^{Mt} \right\|_2 \equiv \left\| e^{Mt_{\max}} \right\|_2, \tag{1.23}$$

which measures the maximum amplitude of an evolved disturbance $\bar{\mathbf{v}}(t)$ (or $v(x,t)$) for all possible initial conditions $\bar{\mathbf{v}}(0)$ (or $v(x,0)$). The initial condition producing the maximal amplification at time $t_{\max}$ is often called the optimal disturbance $\bar{\mathbf{v}}_{\text{opt}}$ and is given by the right singular vector corresponding to the largest singular value of $e^{Mt_{\max}}$ [16]. For normal operators $\gamma = 1$, but for nonnormal ones it can be arbitrarily large. Several authors have introduced quantities similar to (1.23) to characterize transient growth [16, 71–73]. We should point out that the transient amplification factor is analogous to transfer norms which arise in the input-output description commonly used in control theoretic analyses, including those concerning transient growth [2, 44, 50].




(a) (b)

**Fig. 1.1** Control of the Kuramoto-Sivashinsky equation (1.26) using localized feedback applied at the four points marked with circles. (a) Control succeeds for a system of size $l = 55$. Note the strong initial transient localized around the actuators and preceding the asymptotic decay. (b) Control fails for a system of larger size $l = 60$.

Under fairly general assumptions it can be shown [35] that transient amplification does indeed scale with $k_{\text{max}}$:

$$\gamma \sim \frac{k_{\text{max}}}{|\beta'_{\text{max}}|} \propto e^{\frac{l}{Ml_0}}, \qquad \beta'_{\text{max}} = \max_n \beta'_n. \tag{1.24}$$

In case the full information about the system state is unavailable, one has to use an array of sensors to reconstruct it from the local measurements. The duality of this problem to the feedback problem allows us to immediately make a couple of conclusions. First, the array of sensors should be built according to the same principles as the array of actuators to ensure that the system state can be reconstructed. Second, the total transient amplification will be given by the product of those for each stage (sensing, feedback) [25, 34]. If the sensing stage mirrors the feedback stage (same number and arrangement of sensors and actuators and sensing gain equal to feedback gain), we obtain

$$\gamma_{\text{total}} = \gamma^2 = e^{\frac{2l}{Ml_0}}. \tag{1.25}$$

Summing up, we have found that regardless of how small the magnitude $\sigma$ of initial disturbances is, the transient growth in the feedback loop will amplify them to an $O(\gamma\sigma)$ magnitude which, for sufficiently large spacing $s_2 = 2l/M$ between controller pairs, will be large enough for the linear stability analysis to break down and for control to fail. This is illustrated in Fig. 1.1 for the Kuramoto-Sivashinsky equation

$$\partial_t v = -\partial_x^2 v - \partial_x^4 v - v\partial_x v. \tag{1.26}$$

Exactly when the breakdown occurs depends on (i) the magnitude of noise $\sigma$, (ii) the placement of actuators and the choice of feedback gain which affect transient amplification factor $\gamma$, and (iii) the particular form of the nonlinear terms which determines the limits of the validity of the linear approximation. We address this last issue in the next section.

## 1.4 Nonlinearity and the critical noise level

The effect of nonlinear terms can be considered from different perspectives. The simplest argument suggests that, as long as the evolution equations are non-dimensionalized to get rid of very large or very small parameters, the importance of nonlinear terms can be judged based simply on their order of magnitude. We will limit our scope to the most common type of nonlinearities found in spatiotemporal dynamics, those having the form of a power of the disturbance, occasionally with a spatial derivative in the mix, (e.g., quadratic nonlinearities in the logistic coupled map lattice [15,28], Kuramoto-Sivashinsky equation [1], Navier-Stokes equation [48,75] or Boussinesque equations [77], cubic nonlinearities in the CGLE [7] and Swift-Hohenberg equation [35], quartic nonlinearity in thin film equations [27, 60] and so on). An upper bound for the breakdown of the linear control approach is immediately obvious: If a disturbance $\sigma$ is transiently amplified such that $\gamma\sigma = O(1)$, the nonlinear terms become important and the linear approach becomes invalid. This estimate gives the upper bound for the noise level

$$\sigma_{\max} \sim \gamma^{-1}. \tag{1.27}$$

Numerical integration performed for a generalized (real) Ginzburg-Landau equation with a custom nonlinear term $f(v)$,

$$\partial_t v = v + \partial_x^2 v + f(v), \tag{1.28}$$

and with feedback applied at one of the boundaries,

$$v(0,t) = 0, \qquad v'(l,t) = \int_0^l k(x)v(x,t)dx, \tag{1.29}$$

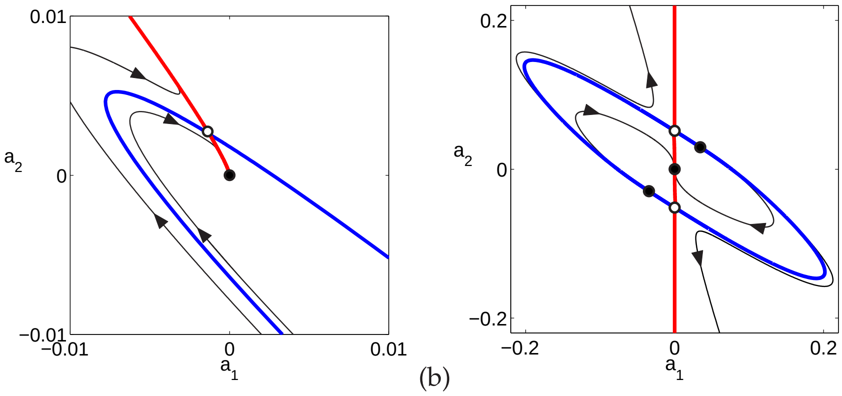
shows that for nonlinearities with an odd power, e.g., $f(v) = v^3$ or $v^5$, one does indeed find the scaling (1.27) at large $l$ [29]. For even powers, e.g., $f(v) = v^2$ or $v\partial_x v$, one instead finds a different scaling law

$$\sigma_{\max} \sim \gamma^\alpha, \qquad \alpha = -\frac{p}{p-1}. \tag{1.30}$$

This scaling can also be understood using order of magnitude arguments and employing the idea of bootstrapping originally introduced by Trefethen et al. [78] in the context of shear flow (in)stability. The idea of the argument is that the purely linear growth leading to the estimate (1.27) is preempted by a positive-feedback loop involving transient amplification and nonlinearity. The critical noise level in this case can be found by equating the order of magnitude of the initial (primary) disturbance with the magnitude of the nonlinear terms acting on the amplified disturbance, which act as a secondary disturbance that is further transiently amplified, $O(\sigma) = O((\gamma\sigma)^p)$. Solving for $\sigma$ one immediately obtains (1.30). The justification of the scaling law for the model (1.28)-(1.29) with an arbitrary power $p$ can be found in [29].

One could ask if the scaling exponents in (1.27) and (1.30) or even the power law scaling itself obtained for a particular model equation are generic and hence our understanding of the effect of nonlinear terms complete. Unfortunately, the answer is negative on both counts. The situation is far more complicated even in the framework of the simple model considered here. One can see this by studying the limit of small, rather than large, system size, as was done in [36]. In this limit all calculations can be done analytically.

Without repeating the details of the analysis we will summarize the results. The system size $l$ is chosen such that only one Fourier mode is unstable and one mode is very weakly stable. Feedback is chosen to make the stable mode weakly stable as well, so that the dynamics of the closed-loop system in the Fourier space is characterized by two slow, nearly degenerate, modes and an infinite number of fast (strongly) stable modes. Adiabatic elimination of the fast modes reduces the dynamics to the subspace spanned by the two slow modes. The analysis performed for the cubic and the quadratic nonlinearity then shows that the basin of attraction of the target state is bounded by the stable manifold of one (for quadratic) or two (for cubic nonlinearity) saddle-type steady states that emerge in the vicinity of the target state as a result of feedback (see Fig. 1.2). The shape and size of the stable manifold determine the critical noise level. Computing the amplification factor $\gamma$ one can find that the power law scaling $\sigma_{\max} \sim \gamma^\alpha$ is an exceptional case. More typically $\sigma_{\max}$




(a) (b)

**Fig. 1.2** The phase portrait of the model (1.28) with the quadratic (a) and the cubic (b) nonlinearity in the subspace parameterized by the amplitudes $a_1$ and $a_2$ of the two slow modes. The filled and the open black dots show the nodes and saddles, respectively. The blue and red curves show the stable and unstable manifolds, respectively, of the saddles. The black curves are the typical trajectories.

is not uniquely determined by $\gamma$, but also depends on the time $t_{\max}$ at which the maximal transient amplification is achieved.

The relation between $\sigma_{\max}$ and $\gamma$ provides the last piece of the puzzle, relating the environmental noise, the symmetry of the system, the density of the sensor/actuator array, and the choice of the closed-loop eigenvalues through equations such as (1.22), (1.24), and (1.30).

## 1.5 Conclusions

The field of feedback control of nonlinear spatially extended systems has grown too large in the past ten or so years to give credit to all researchers who have contributed to its development. In this chapter, we discussed some of the recent results, concentrating mostly on localized feedback control. >From the discussion presented in these pages it should be clear that our understanding has reached a level of maturity necessary to address real problems of interest.

On the other hand, many problems remain unresolved. For instance, the feedback control of spatially and temporally periodic states has received much less attention than control of uniform steady states, with numerical studies overwhelming analytical investigations. The nonlinear stability of closed-loop systems is another area where progress has been limited, with the majority of studies concentrating on low-dimensional models rather than true spatiotemporal dynamics. Another fundamental problem awaiting solution is the problem of "targeting", as it is referred to in the context of low-dimensional systems, which becomes progressively more challenging as the dimensionality of the system increases.

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